How to Design a PDN for Worst Case?

Istvan Novak, Oracle, December 2015

In the previous column [1] we showed that for Linear and Time Invariant (LTI) systems the *Reverse Pulse Technique* [2] is a simple, fast and guaranteed way to obtain worst-case transient noise. The worst-case excitation and its resulting noise wave, sometimes called rogue wave, could also be approximated by speculative waveforms, but potentially with errors. We showed that when applied properly, the target impedance concept is a useful and valid design tool for power distribution networks (PDNs), but the question remains: what should be the design process to account for worst-case noise. In this column we get the answer.

As a starting point, we briefly summarize here what we learned in the previous column. We used a circuit from [2], shown in *Figure 1*, with an impedance profile shown in *Figure 2*. This circuit has three anti-resonance peaks: 67 kHz, 1 MHz and 51 MHz. The resonance peaks all have approximately 100 mOhm impedance magnitudes. These peaks are clearly visible not only on the impedance plot, but also on the *Step Response* plot in *Figure 3*, we just need to switch the horizontal scale to logarithmic. From the *Step Response,* we can apply the *Reverse Pulse Technique* and get the absolute worst-case transient noise, 391 mVpp, which is shown in *Figure 4*. The *Step Response* has a peak deviation of 29.6 mV, which together with the 3mV DC steady state response on the 3 mOhm DC resistance creates a 56.2 mVpp worst-case noise estimate. We get this amount of noise when a positive-going 1A current step is followed by a 1A negative-going current step with sufficient time between the two current steps so that the response can settle to its steady state before the next step arrives. In contrast, from the straight target-impedance calculations we would expect 100 mV noise.

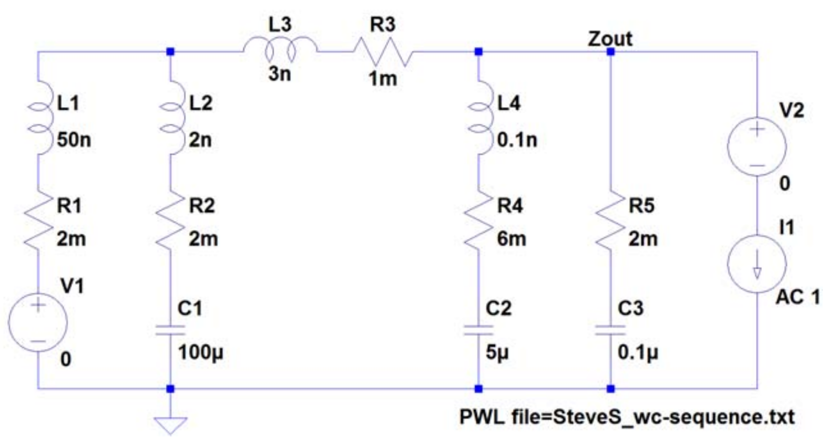

Figure 1: *Rouge-wave example circuit from [2].*

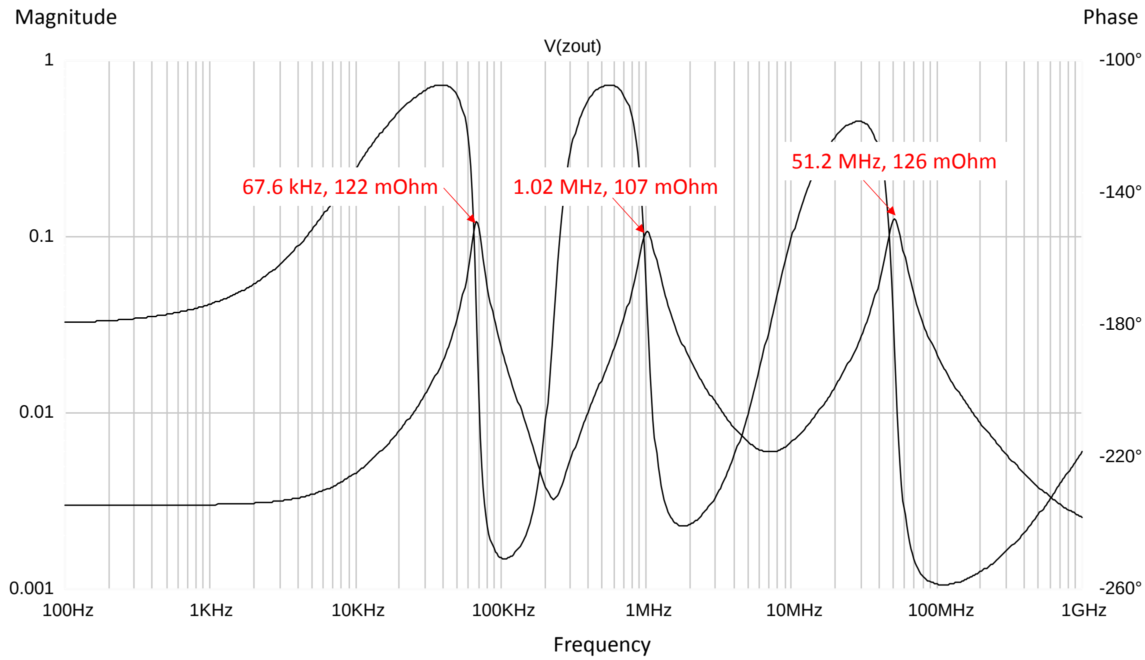

Figure 2: *Impedance magnitude and phase from the circuit shown in Figure 1. Note that both axes are logarithmic; in particular, the frequency scale is logarithmic to clearly show the resonance peaks separated by three orders of magnitude.*

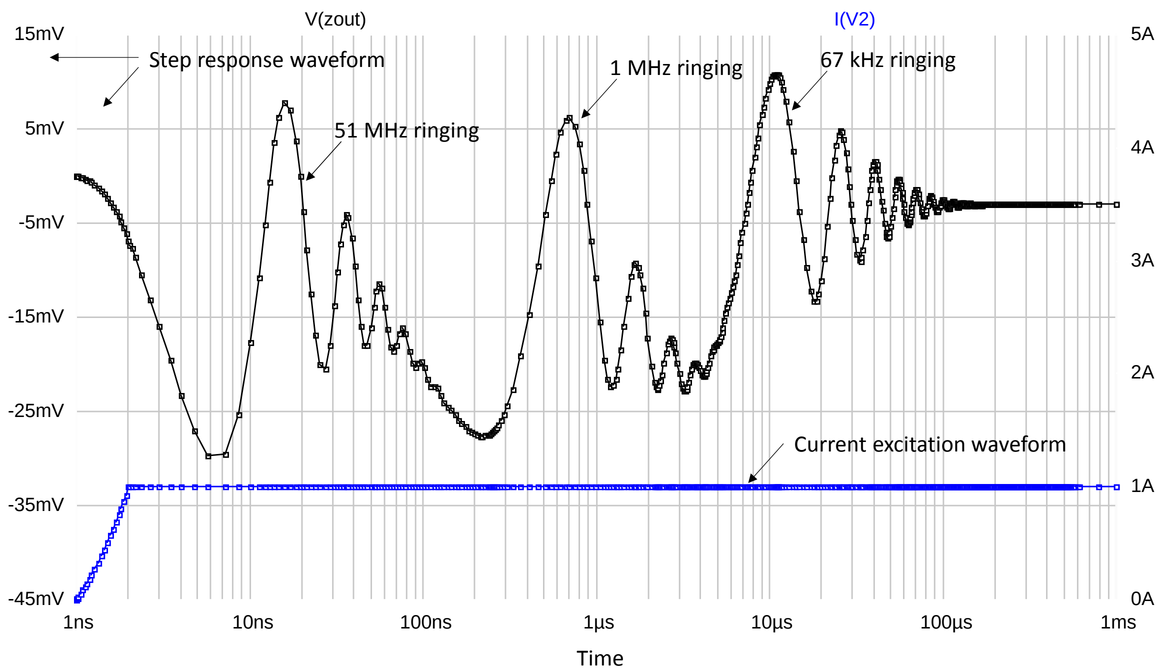

Figure 3: *Simulated Step Response of the circuit shown in Figure 1. Vertical axis is linear, the horizontal axis is logarithmic.*

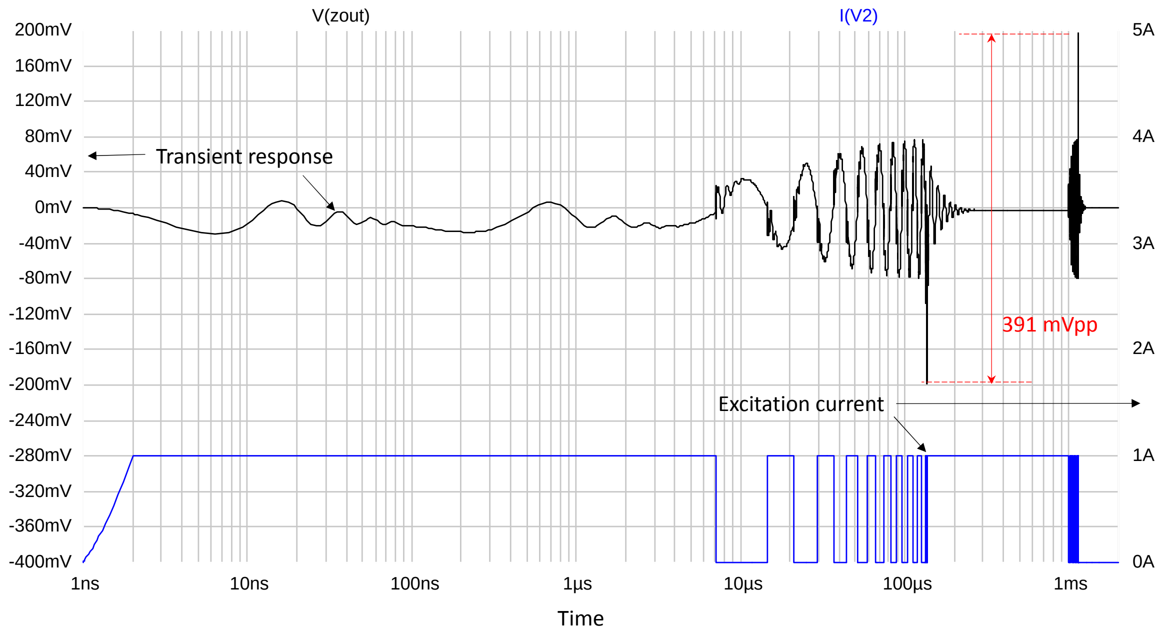

Figure 4: Worst-case response simulated with an excitation sequence calculated from the Reverse Pulse Technique.

In worst case, however, when the positive and negative-going 1A current steps can hit the circuit in any arbitrary sequence, the *Reverse Pulse Technique* on *Figure 4* predicts a 391 mVpp maximum noise, more than six times higher than what we get from the peak deviation of the Step Response.

In this column we will look at a few further cases illustrating what happens when we have different degrees of 'non-flatness'.

When we have a linear network, the excitations and the impedance profiles can be scaled, so it does not matter what impedance target we use for the illustrations. For sake of simplicity and consistency, we will use a 100 mOhm impedance target and for all examples we will make sure that within the bandwidth of the excitation, the impedance does not exceed this limit.

Figure 5 shows the impedance profiles of four cases. We start with a single peak at 0.1 MHz. A second peak is added at one and half decade higher, at 3.16 MHz, also with exactly 100 mOhm peak value. The third peak is added one and half decade below the first resonance, at 3.16 kHz. Finally a fourth peak is added at one and half decade above the second peak, at 100 MHz. Note that at very low and very high frequencies the impedance settles at 1 mOhm, 1% of the peak value. The one and half decade separation between the peaks allows the impedance magnitude to drop substantially in between, close to the 1 mOhm asymptote values.

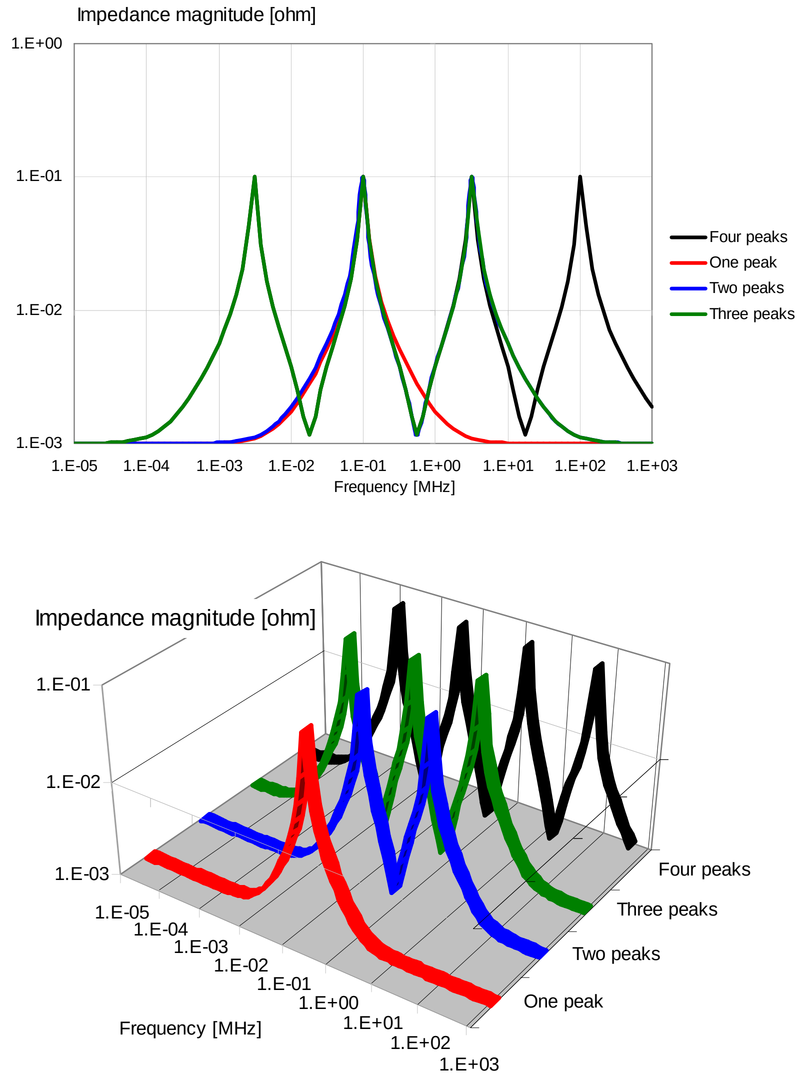

Figure 5: *Impedance profiles with one, two, three and four distinct peaks, all reaching exactly 100mOhm values. The top and bottom plots show the same data: on the top chart we can better see that all four peaks reach exactly 100 mOhm values. The bottom chart shows better how the peak frequencies in the four cases relate to each other.*

Figure 6 shows the *Step Response* of each of the four cases. Note that the horizontal scale is logarithmic to accommodate the ringing of widely differing frequencies. All four cases have impedance profiles not exceeding a 100-mOhm target value, so ignoring the non-flatness of the impedance, one would expect 100 mVpp worst-case transient noise. Instead, based on the *Reverse Pulse Technique*, we get 120, 234, 346 and 453 mVpp worst case values. The biggest hit is the initial factor of two increase; as we showed it in [1], this happens because instead of a flat impedance starting at DC with the target impedance value, we start with zero (or very low) impedance and then continue with a flat target impedance at higher frequencies. When we have just one dominant peak, reaching the target impedance at the peak, but having very low impedance at DC and at high frequencies, we create a the bandpass filter. This produces the worst-case noise when we repetitively hit this peak with a 50% duty cycle square wave. The bandpass filter picks out the fundamental harmonic from the square wave, creating a 4/PI times higher response.

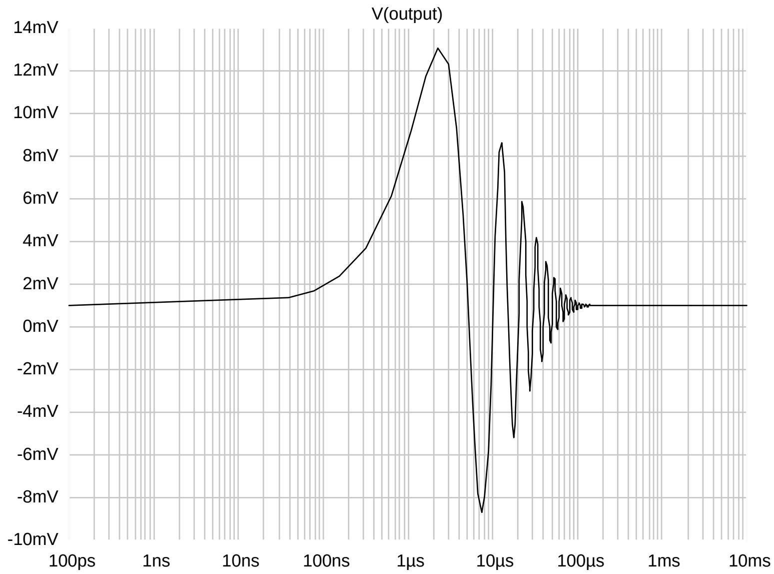

Figure 6a: *Step Response with one 100 mOhm peak. Worst-case transient noise from the Reverse Pulse Technique is 120 mVpp for each ampere of excitation.*

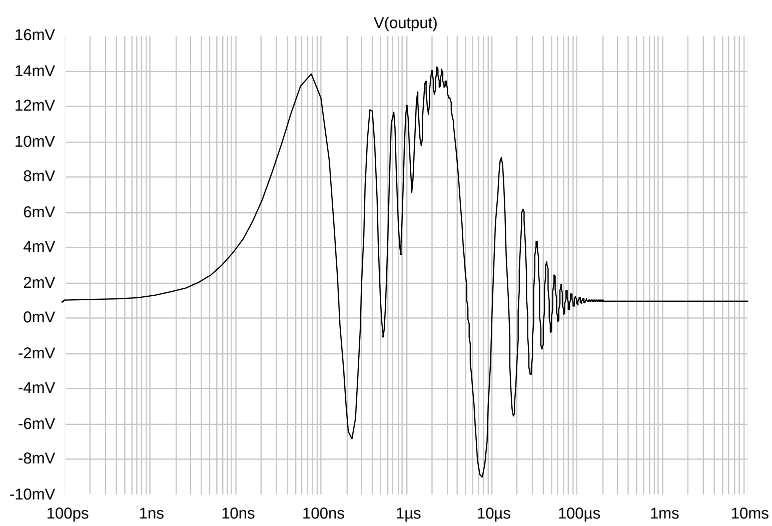

Figure 6b: *Step Response with two 100 mOhm peaks. Worst-case transient noise from the Reverse Pulse Technique is 234 mVpp for each ampere of excitation.*

As the number of resonant peaks increase in the impedance profile, the worst-case noise goes up. In the example shown here, the peaks are fairly well separated on the frequency scale, interacting only mildly. The small interaction reduces somewhat the worst-case peak noise from the pathological worst case of 120, 240, 360, 480 mVpp values that we get when the peak responses do not interact.

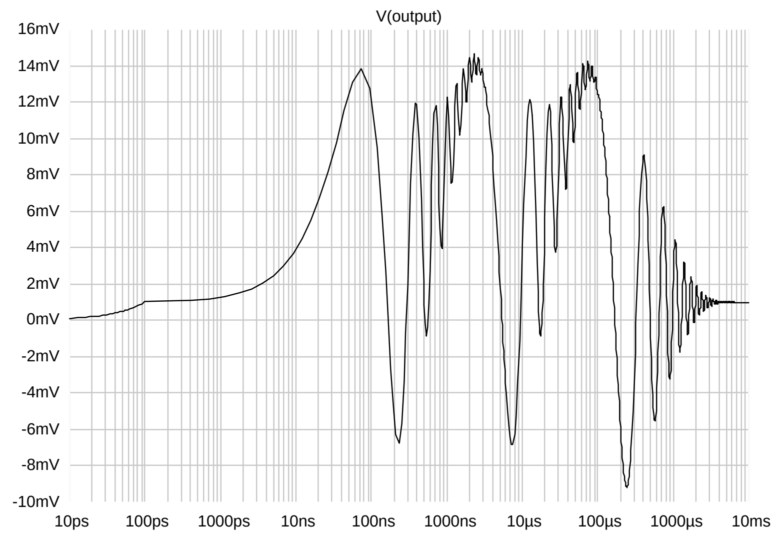

Figure 6c: *Step Response with three 100 mOhm peaks. Worst-case transient noise from the Reverse Pulse Technique is 346 mVpp for each ampere of excitation.*

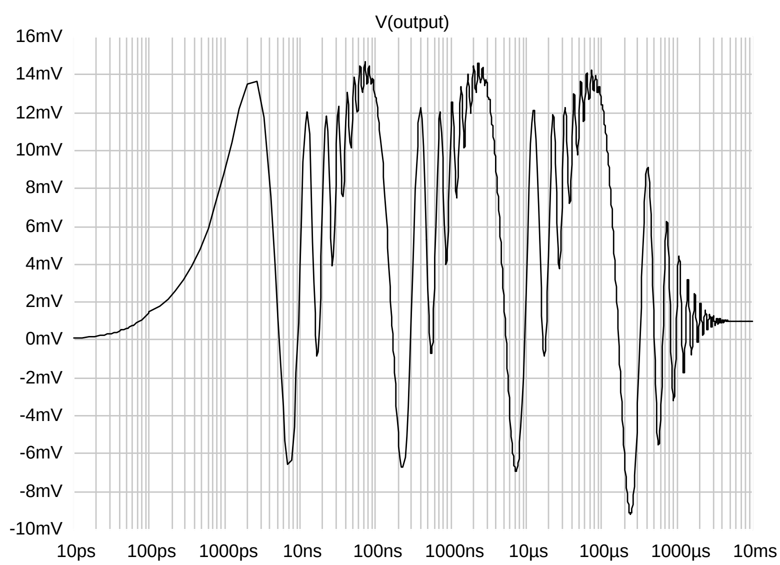

Figure 6d: *Step Response with four 100 mOhm peaks. Worst-case transient noise from the Reverse Pulse Technique is 453 mVpp for each ampere of excitation.*

Next we look at a single disturbance in a flat impedance profile. We use the same 100 mOhm target impedance as before and drive a deep second-order notch into it with three different Q values: 1, 3 and 10. *Figure 7* shows the impedance profiles, *Figure 8* shows the *Step Responses*. Note that all three responses reach a 1 mOhm minimum impedance at 1 MHz.

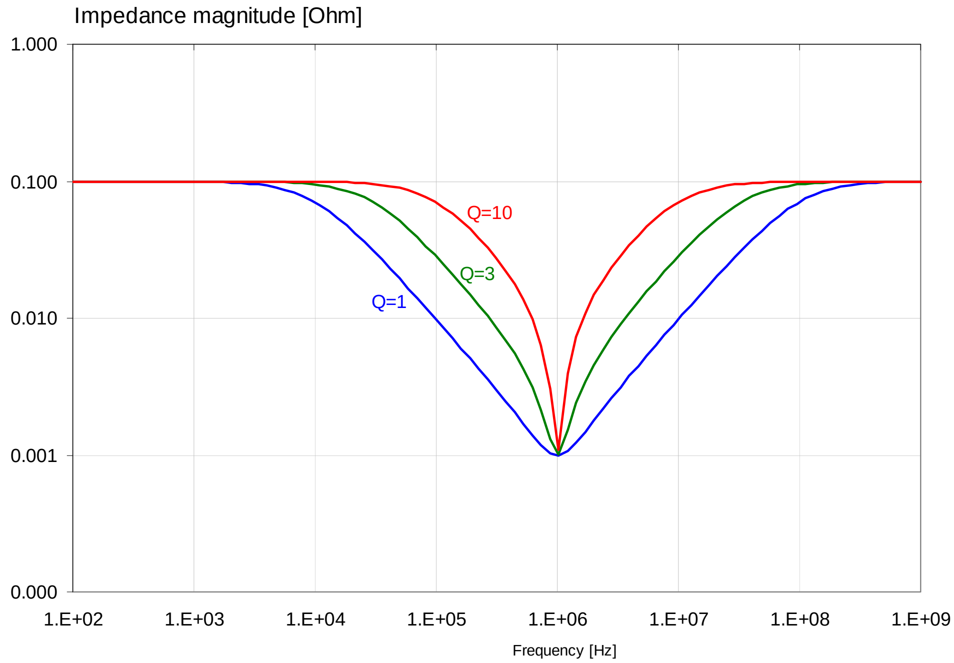

Figure 7: *Flat impedance profile with a single second-order notch at 1 MHz frequency.*

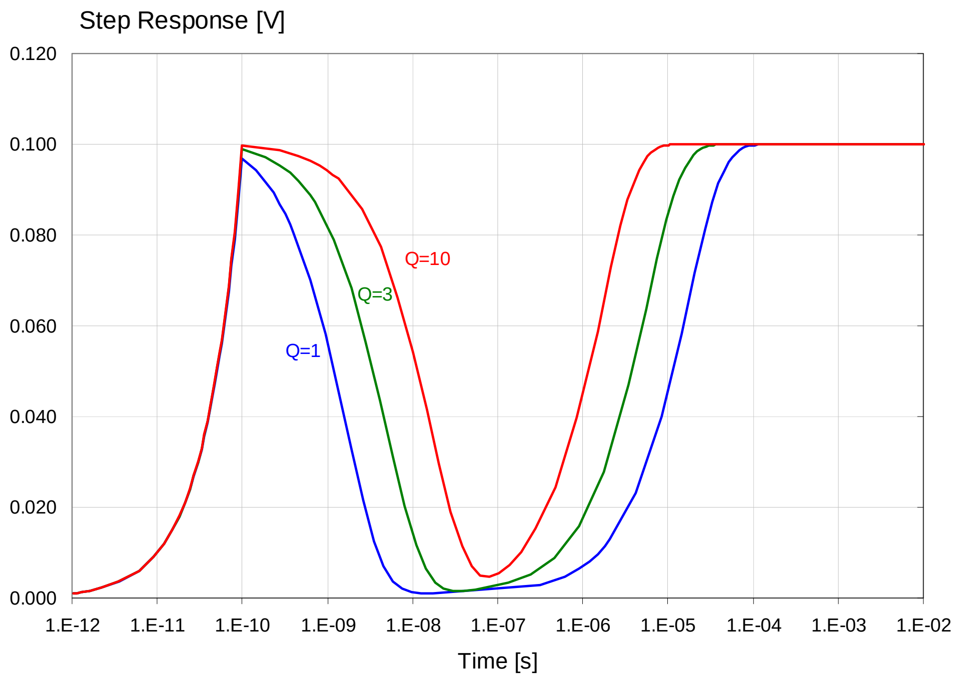

Figure 8: *Step Responses of circuits from Figure 7.*

Interestingly, for a single disturbance in the impedance profile with the same maximum and minimum values, the worst-case transient noise does not depend on the Q of the notch. When we calculate the worst-case noise with the Reverse Pulse Technique, we get 290mVpp for all three cases. *Figure 9* shows the actual worst-case time-domain response for the Q=10 case.

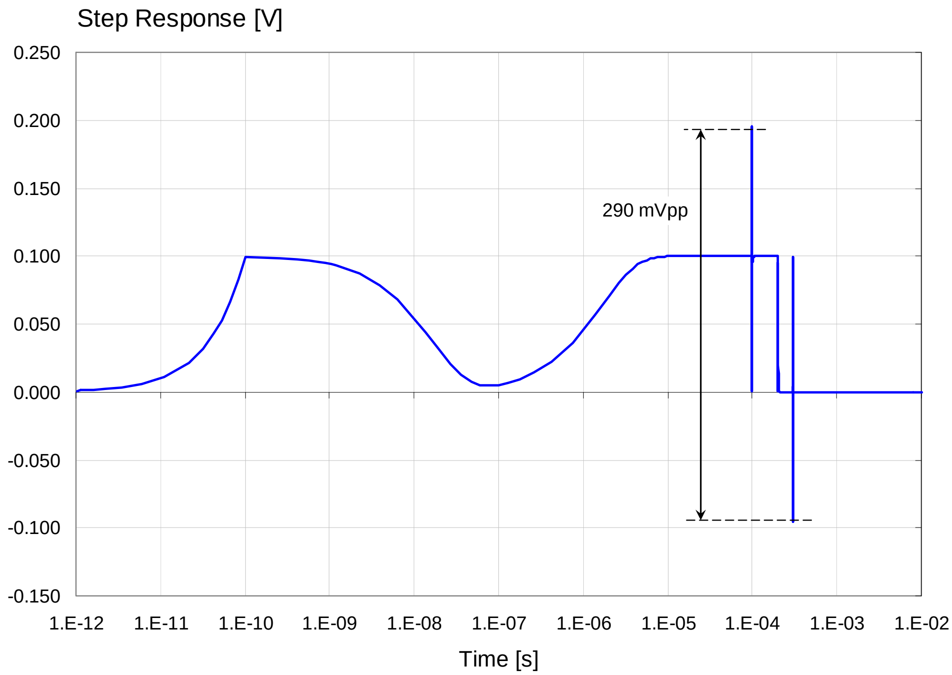

Figure 9: *Worst-case transient peak-to-peak noise with 1A 100ps rise time step excitations. Q=10. Note the logarithmic horizontal scale.*

Note the enormous increase of noise: from the 100mVpp value for a perfectly flat impedance, the noise went up almost three fold, even though we stay within the impedance target!

Lastly we show the noise penalty as a function of notch depth. We already showed that the Q value is irrelevant, so we use an arbitrary Q=3 value and set the second-order notch to produce an impedance minimum at 1 MHz with a series of values between no notch (100 mOhm) and 1 mOhm. The impedance profiles are shown in *Figure 10*, the *Step Responses* are shown in *Figure 11*. The worst-case transient noise for 1A step excitations is shown in *Figure 12*.

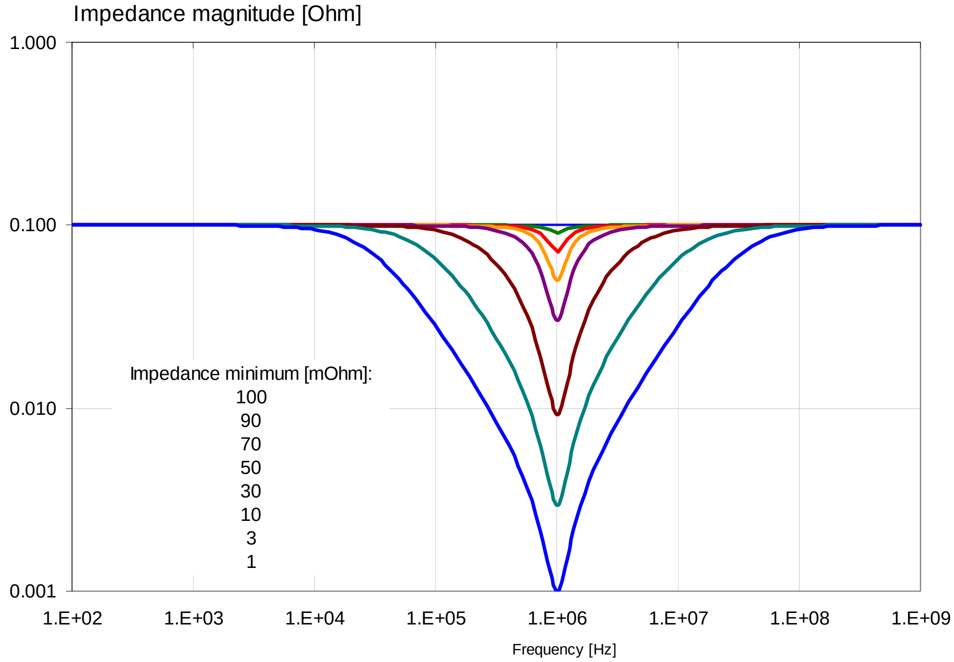

Figure 10: *Magnitude of a flat impedance with a single second-order notch with different minimum values at 1 MHz.*

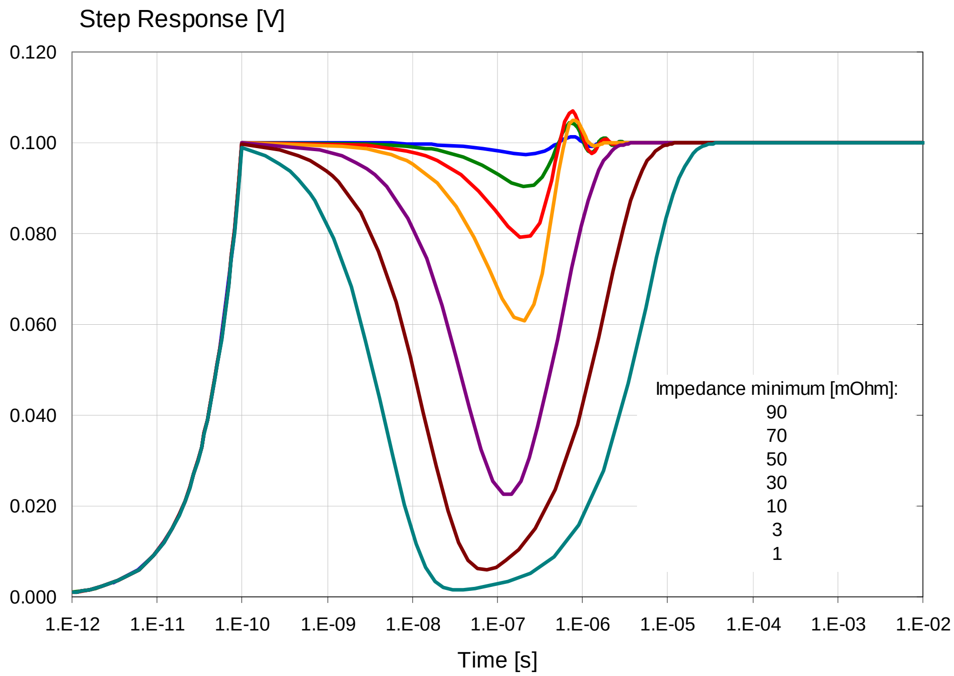

Figure 11: *Step Responses of the flat impedance profiles with a single second-order notch with various minimum impedance values from Figure 10.*

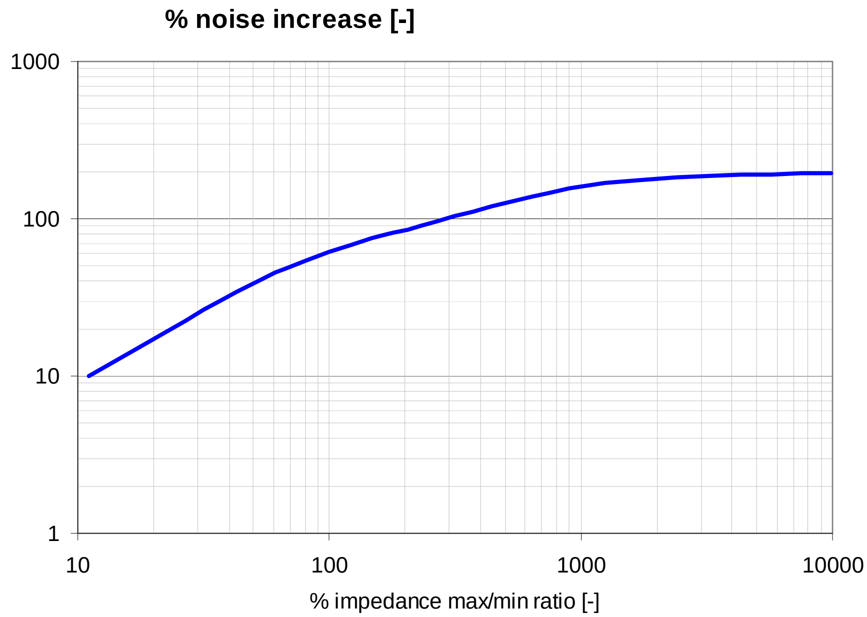

Figure 12: *Relative noise increase as a function of relative max/min ratio of impedance profile on a flat impedance with a single second-order notch.*

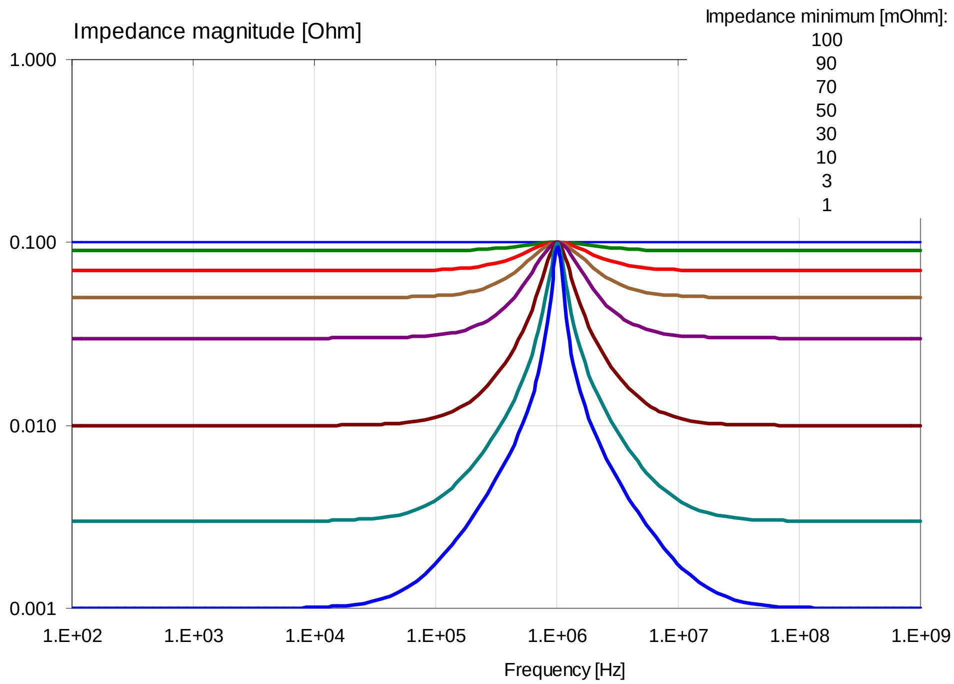

Figure 13: *Magnitude of a flat impedance with a single 100 mOhm peak at 1 MHz with different minimum values.*

Figure 12 clearly shows the penalty of a non-flat impedance profile: for small deviations it varies linearly and proportionally to the max/min impedance ratio.

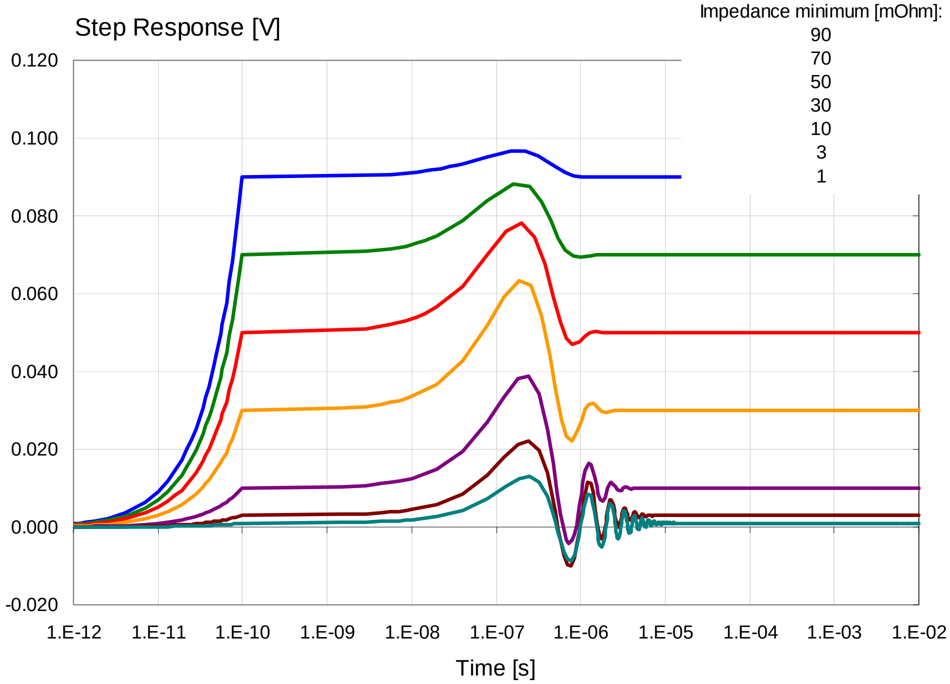

Figure 14: *Step Responses of the impedance profiles from Figure 13.*

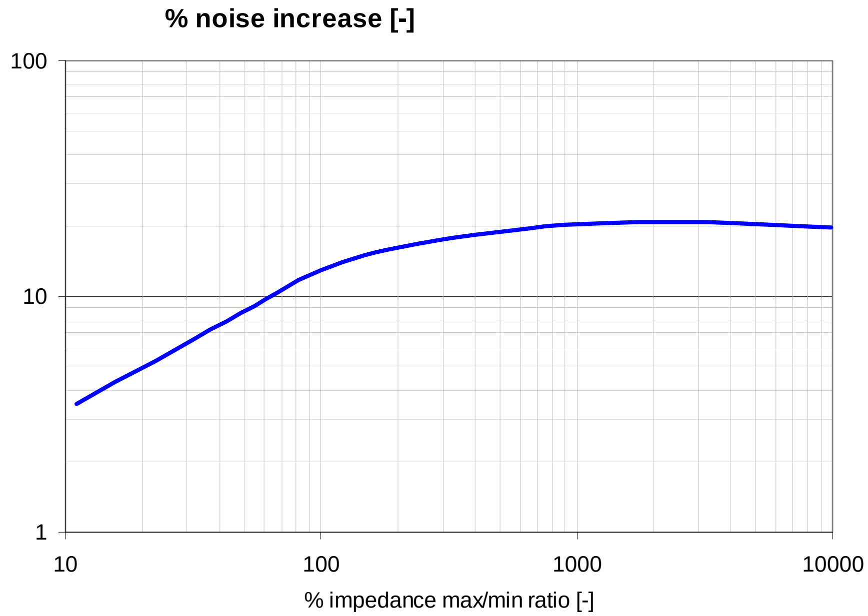

Figure 15: *Relative noise increase as a function of relative max/min ratio of impedance profile.*

For a single second-order notch with large deviations, the noise penalty saturates at about 3x. All the above means that very counter-intuitively noise goes up substantially even if we just push the impedance down at certain frequencies, even if we stay within the pre-defined maximum.

Lastly we flip around the impedance profile and use one peak fixed at 100 mOhm maximum value, and we vary the value of low-frequency and high-frequency asymptotes. This essentially creates the inverse of impedance profiles we had in *Figure 10*: it was a band-reject function there, now we look at a pass-band function. *Figure 14* shows the corresponding *Step Responses*. Finally *Figure 15* shows the percentage penalty as a function of max/min impedance ratio.

Note these cases do not intend to represent practical scenarios, they merely serve our better understanding. In practice it is very unlikely to have multiple impedance peaks or notches with the same extreme values. Nevertheless these examples serve as a guidance for the design process. If we use the target impedance approach and assume that due to non-flatness the worst-case noise is approximately three times higher, we can readjust our impedance target and we can then do a straightforward design process.

For more information on the subject, you can check out [3] and [4].

References:

[1] Systematic Estimation of Worst-Case PDN Noise: Target Impedance and Rogue Waves, QuietPower column, December 2015

[2] Steve Sandler, “Target Impedance Limitations and Rogue Wave Assessments on PDN Performance,” paper 11-FR2 at DesignCon 2015, January 27 – 30, 2015, Santa Clara, CA.

[3] Target Impedance and Rogue Waves, panel discussion at DesignCon 2016, January 19 – 21, 2016, Santa Clara, CA.

[4] Jae Young Choi, Ethan Koether, Istvan Novak, “Electrical and Thermal Consequences of Non-Flat Impedance Profiles,” DesignCon 2016, January 19 – 21, 2016, Santa Clara, CA.