

Short Tutorial on Quartz Crystals and Oscillators

Contents

| | |
|--|----|
| 1. Quartz Crystals | 2 |
| 1.1 Equivalent circuit of a quartz crystal | 2 |
| 1.2. Quartz crystal in 'series resonance' | 5 |
| 1.2.1. Influence of the shunt capacitance | 6 |
| 1.2.2. Influence of high frequencies..... | 7 |
| 1.2.3. Series resistance, motional capacitance, quality factor | 7 |
| 1.3. Quartz crystal aligned with load capacitance | 9 |
| 1.4. Unwanted resonances of quartz crystals | 11 |
| 2. Quartz Crystal Oscillators | 12 |
| 2.1 Pierce oscillator with inverting amplifier | 12 |
| 2.2 Colpitts oscillator with non-inverting amplifier | 14 |
| 2.3 Colpitts oscillator with bipolar transistor | 15 |

1. Quartz Crystals

1.1 Equivalent circuit of a quartz crystal

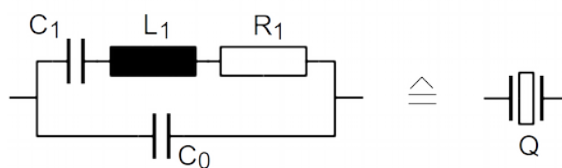


Figure 1
Equivalent circuit of a quartz crystal

The mechanical oscillation, piezo-electrically excited by an alternating electrical field of suitable frequency, corresponds with the above equivalent circuit, which consists of a series resonant circuit together with a capacitance in parallel. Usual parameters are the motional capacitance C_1 , the motional inductance L_1 , the motional resistance R_1 , representing the so-called motional (dynamic) branch, and the shunt (static) capacitance C_0 .

The motional capacitance C_1 represents the mechanical elasticity and the motional inductance L_1 corresponds to the mechanical mass. The motional resistance R_1 combines losses of internal friction, dampening effects of the surrounding atmosphere and the mounting arrangement. Finally, the electrode areas on the crystal surface are responsible for most of the static capacitance C_0 .

On the basis of this equivalent circuit the series resonance f_s of the motional branch with C_1 and L_1 can be defined, as well as the so-called parallel resonance f_p of the circuit consisting of C_0 and, in this case, the inductive branch of C_1 and L_1 .

The **series resonant frequency** f_s of the quartz crystal is defined by the series resonance of the motional branch

$$f_s = \frac{1}{2\pi\sqrt{C_1 L_1}} \quad (1)$$

The **parallel resonant frequency** f_p is defined by

$$f_p = \frac{1}{2\pi\sqrt{\frac{C_1 C_0}{C_1 + C_0} L_1}} = f_s \sqrt{1 + \frac{C_1}{C_0}} \quad (2)$$

$$\approx f_s \left(1 + \frac{C_1}{2C_0}\right) \quad \text{for } C_0 \gg C_1 \quad (2a)$$

For both frequencies the crystal impedance is not purely resistive, e.g. at f_s by the shunt C_0 .

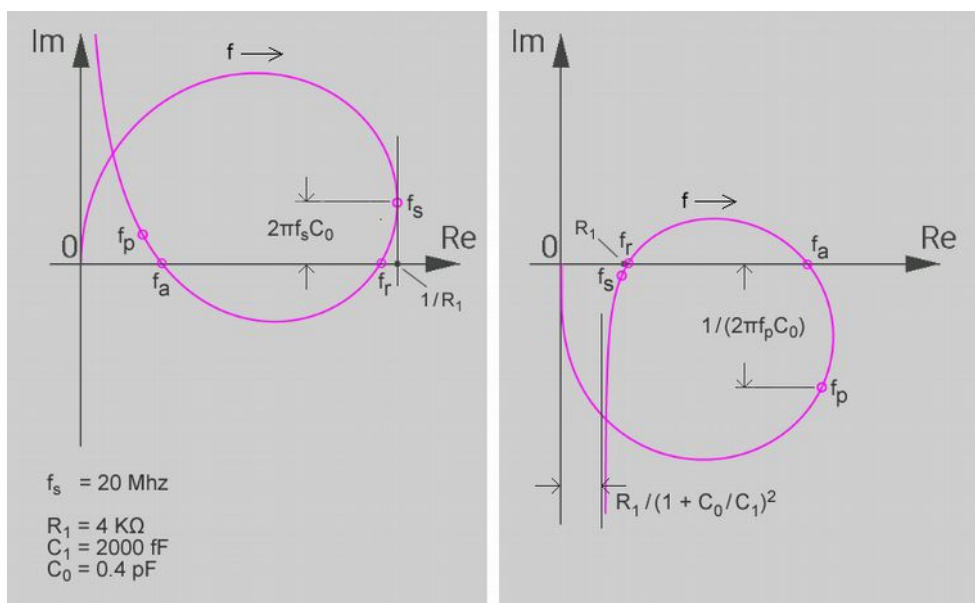
Figures 2 and 3 show the impedance and the admittance corresponding to the equivalent circuit in Figure 1. f_s and f_p are marked with little dots on the curves (f_p according to equation 2, not 2a!). However, these curves correspond to component values, which can not be realized with normal quartz crystal - but they can illustrate in principle the basic location and sequence of the characteristic frequency points. With normal quartz crystals the high quality factor prevents important points of the diagram to be shown completely without unrealistic deformation of scaling and curves. Figure 4 shows

the admittance of a normal quartz crystal. The curve is nearly symmetric in respect to the real axis; the series resonance is very near to the point $1/R_1$. Using the impedance diagram (Figure 5), f_s and other important points would lie unidentifiable near zero, if one included the far right parallel resonance to scale. Therefore, only the important low impedance values are depicted.

The frequency where the impedance is real and of low resistance, is called **resonant frequency f_r** (without the adjunct 'series!'). This important and easily measurable crystal parameter is, at least at not too-high crystal frequencies, only minimally higher than f_s , which plays a minor role e.g. when ordering a crystal.

This is equally valid with regard to the parallel resonant frequency f_p : a reactive component exists at the parallel resonant frequency f_p . The frequency where the crystal impedance is real (and of high resistance) is the **anti resonant frequency f_a** which is somewhat lower than f_p .

In Figures 2 and 3 the Nyquist diagrams of admittance and impedance for the above introduced equivalent circuit are shown. These diagrams are calculated with component values that are not realizable with 'normal' quartz resonators. But they are meant, as has already mentioned, to show the basic location and sequence of important points.

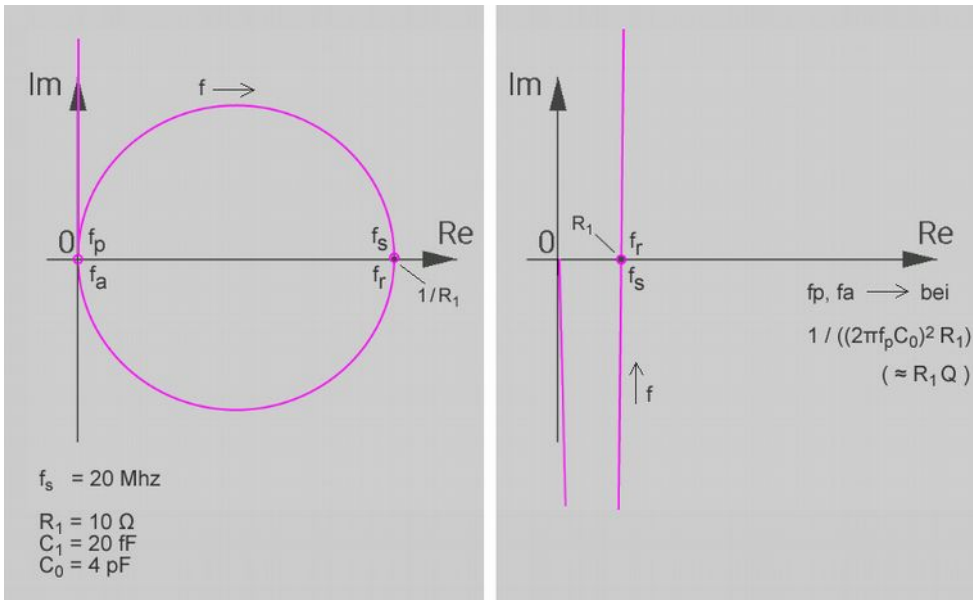


Figures 2 und 3
Admittance (left) and impedance (right) of the equivalent circuit of Figure 1

The next two graphs show the results for a 'normal' quartz resonator.

The loops in Figures 2 and 3 have (almost) become circles in Figures 4 and 5 (next page) and the characteristic frequency points are very close together. The 'circle' in an impedance diagram cannot be shown completely in a diagram that displays the important low impedance points in a recognizable manner. In Figure 5, therefore the corresponding arc is almost a straight line.

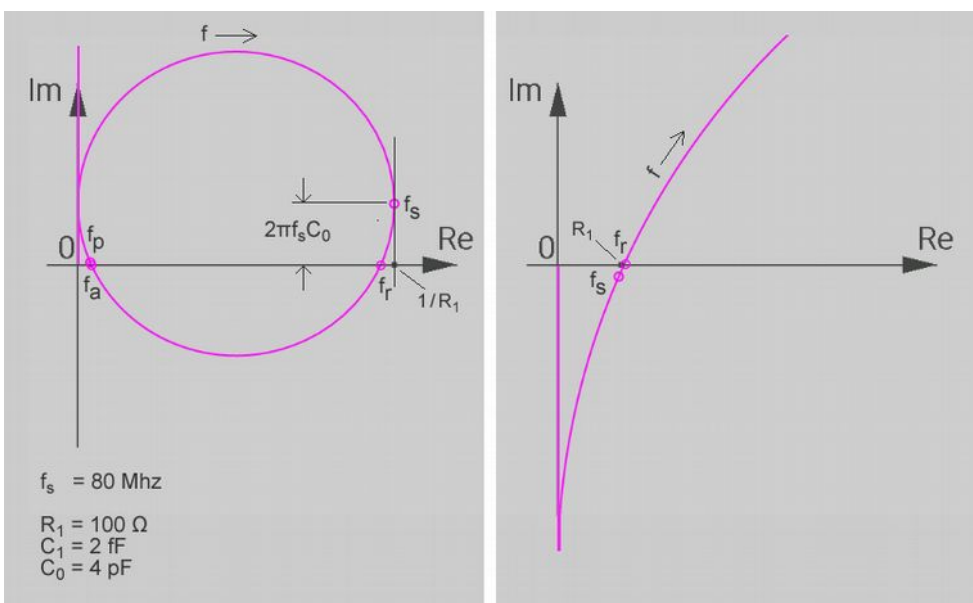
Note: In order to make the following diagrams 'complete' parts of the curves have been added manually (straight lines for lower and higher frequencies outside the actually calculated analyzing range).



Figures 4 and 5

Admittance (left) und impedance (right) of a "normal" quartz resonator

The next two Figures, 6 and 7, represent a higher frequency resonator, for which the graph is clearly asymmetric with respect to the real axis. After all, one can imagine a 'circle' even in the impedance representation. The locations of f_r and f_s are clearly visible.



Figures 6 and 7

Admittance (left) und impedance (right) of a resonator with higher frequency

1.2. Quartz crystal in 'series resonance'

'Series resonant quartz crystal' means alignment of the resonator to the desired frequency during production without additional series capacitance. This commonly used expression is slightly out of focus, because it will not be aligned to the series resonance frequency of the dynamic branch, but to a frequency where the impedance is real: the resonance frequency f_r .

Note: Series resonant quartz crystals are not the normal case, crystals are much more often used with load capacitance. More on that later.

The diagrams resp. simulations shown so far (and the subsequent ones too) are made with the Android app **Y-QUARTZ** by GEYER ELECTRONIC. A couple of pictures have been enhanced a bit with a graphics program.

The default (reset) values for the 'Crystal' structure represent a quartz crystal aligned with the load capacitance which might be used e.g. in a voltage controlled oscillator (VCXO or Voltage Controlled X-Tal Oscillator).

But let us first examine the quartz crystal without load capacitance. Under 'Crystal' we have to choose the 'Series resonance' mode. The resonator is thus simulated as a series resonant resonator: a resonator aligned by the manufacturer without using extra load capacitance.

Under 'Structure' we set the slider for the user's circuit or load capacitance C_L to the far right to ' ∞ ', to replicate a non-effective, i.e. shorted circuit capacitance. As Nyquist diagram we choose 'Admittance'. The admittance plane is more suitable to display the Nyquist graph, because interesting low impedance points are clearly visible even when the full circle is being shown. Furthermore, the influence of C_0 as an additive component of the admittance is somewhat clearer in this presentation.

We simulate a quartz crystal as described in the equivalent circuit with the parameters C_1 , L_1 , R_1 (the so-called 'motional branch') and the 'static' shunt capacitance C_0 without any series (load) capacitance. In Figure 8 we see (as red tips near $1/R_1$ and 0 in the Nyquist diagram) the low-impedance resonance frequency f_r and the high-impedance anti resonance f_a .

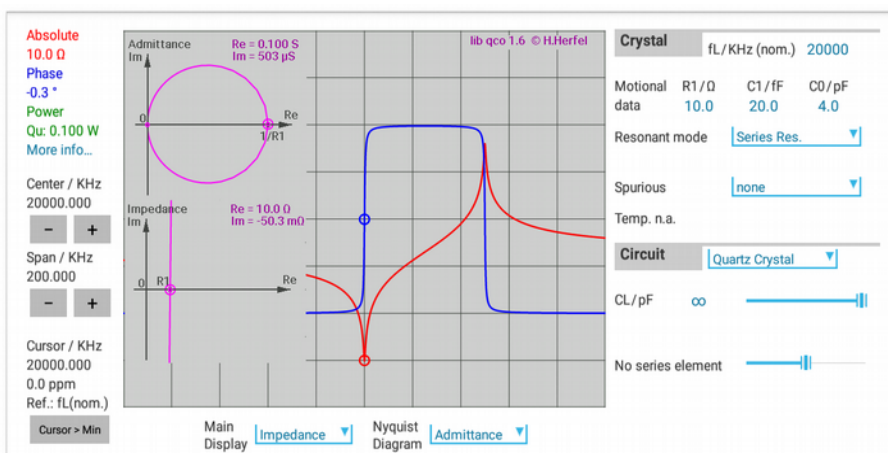


Figure 8

Admittance and impedance of a series resonant resonator

The quartz crystal shown has a nominal resonance frequency of 20 MHz, a motional resistance R_1 of 10 Ω , a motional capacitance C_1 of 20 fF (femtoFarad, 1 fF = 10^{-15} Farad) and a shunt capacitance C_0 of 4 pF (picoFarad, 1 pF = 10^{-12} Farad).

The main display shows magnitude and phase response over frequency. The two resonances f_r and f_a are identified by the zero crossings of the phase near to the two resonance peaks. In addition to the Nyquist admittance, the diagram shows the Nyquist impedance graph, at least the low impedance part of the too-big circle. The 'circles' for this not very high frequency resonator (20 MHz) are in both representations almost symmetrical to the real axis. The arc below the real axis of the Nyquist admittance diagram corresponds to the section between the two extreme values of the red magnitude curve of the main impedance diagram. There, the quartz crystal is 'inductive', the phase of the admittance is negative (positive for the impedance). Outside of this range the quartz crystal behaviour is 'capacitive'.

1.2.1. Influence of the shunt capacitance

If we increase the simulated value of the shunt capacitance C_0 to a (not very realistic) value of 40pF, the Nyquist graph become to lie somewhat higher. In addition, the high-impedance resonance f_a moves closer to the low-impedance frequency f_r . This effect corresponds to the now smaller term $C_1 / 2C_0$ in equation 2.

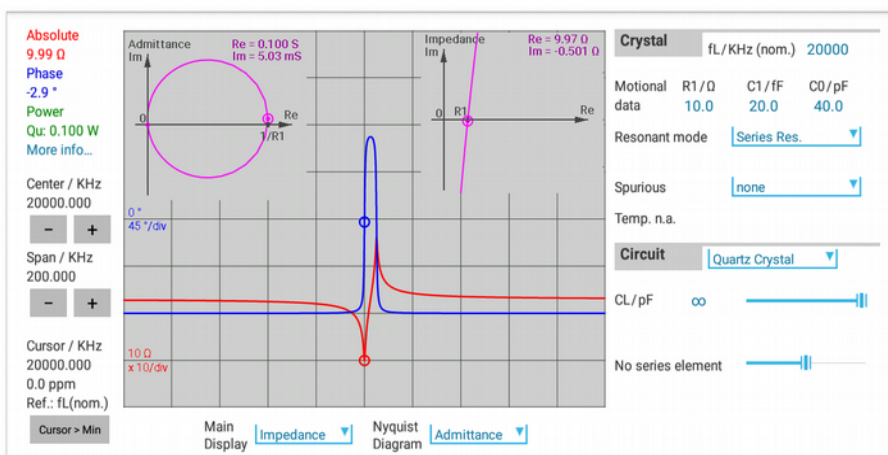


Figure 9
Series resonant resonator
with extremely large
(unrealistic) C_0

1.2.2. Influence of high frequencies

Also at very high frequencies resonators the 'circles' often lie much higher; in certain circumstances so high that they no longer intersect the real axis. An inductive C_0 -compensation can provide a remedy in such cases, both in production during alignment as well as in the application circuit.

Figure 10 (next page) shows as marked frequency values on the Nyquist admittance diagram the sequence of maximum impedance-frequency (cursor), the series resonant frequency f_s (right small purple dot) and resonance frequency f_r (right intersection with the real axis).

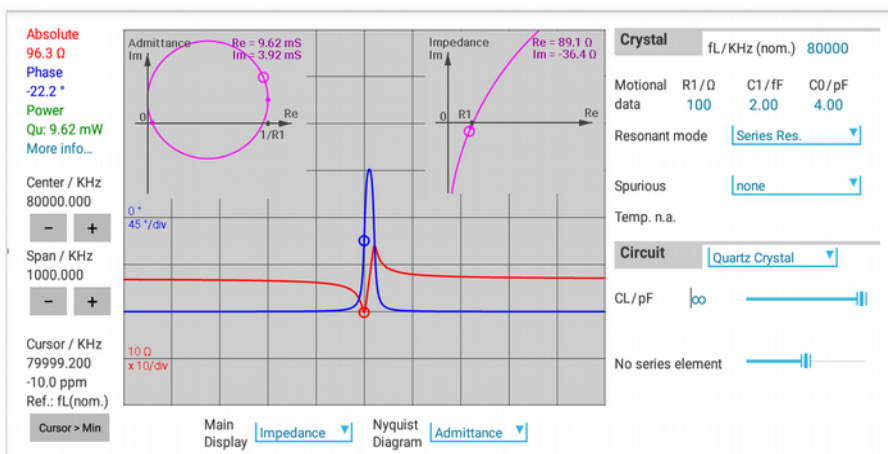


Figure 10
Series resonant resonator with high frequency

For the following analysis of the resonator quality factor we reset the equivalent parameters of the resonator to the values $R_1 = 10 \Omega$, $C_1 = 2 \text{ fF}$, $C_0 = 4 \text{ pF}$.

1.2.3. Series resistance, motional capacitance, quality factor

The two major advantages of quartz crystals are very good frequency stability over temperature and the high quality of the oscillation, i.e. the sharpness of the resonance.

The quality factor Q can be expressed with a formula including the motional resistance R_1 and the motional capacitance C_1 or the motional inductance L_1 :

$$Q = \frac{1}{2\pi f C_1 R_1} = \frac{2\pi f L_1}{R_1} \quad (3)$$

Higher C_1 as well as higher R_1 mean reduction to the quality factor. In addition higher C_1 results in a bigger distance between series and parallel resonance.

Figure 11 (next page) shows the graphs for $R_1 = 10 \Omega$ and $R_1 = 50 \Omega$ superimposed. The resonance peaks are flattened with R_1 being bigger: The quality factor is lower.

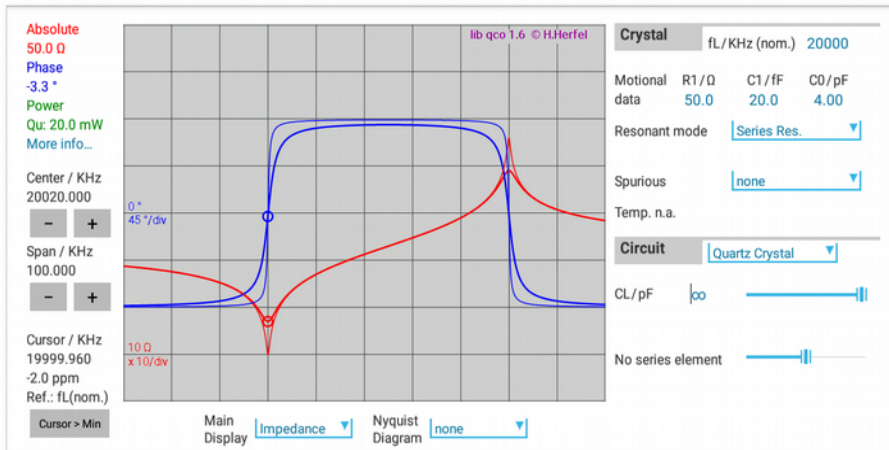


Figure 11
Influence of R_1
(Thin lines: $R_1 = 10\Omega$)

With bigger motional capacitance C_1 , the quality factor would also get lower, but to stay realistic with the C_1 value, the following Figure 12 shows the effect of a four time smaller C_1 of 5 pF: The resonances are sharper, the quality factor increases.

However, we also see that the distance serial / parallel resonance is reduced according to the reduction of C_1 by a factor of 4 (see Equations 2 and 2a).

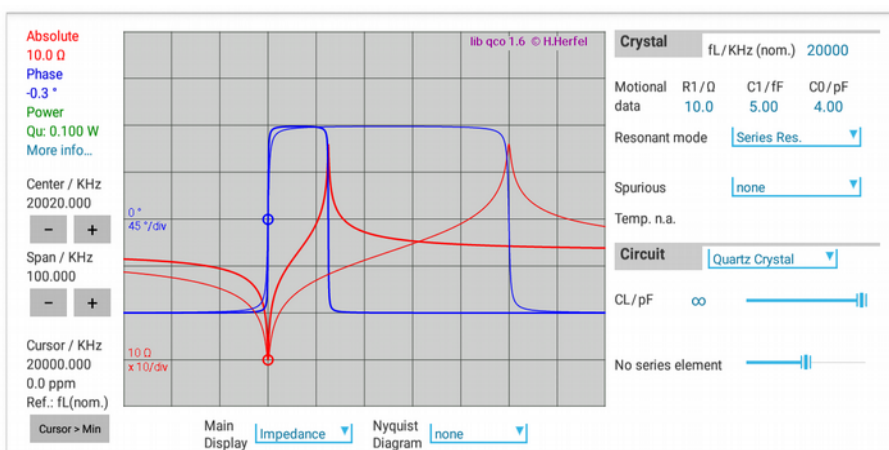


Figure 12
Influence von C_1
(Thin lines: $C_1 = 20fF$)

1.3. Quartz crystal aligned with load capacitance

As mentioned before, the usual requirement is a quartz crystal with load capacitance. The reason is simple: oscillator circuits generally offer a capacitive load component to the resonator at its connection points. Usually this is due to capacitors ensuring oscillation as parts of the feedback network of an oscillator circuit. The load capacitance C_L used in the manufacturing process or the alignment procedure of the manufacturer must then comply with this capacitance in the user circuit and is therefore an important specification parameter together with the **load resonant frequency f_L** , which is defined as the low impedance resonance of the circuit in Figure 13.

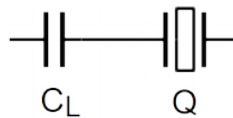


Figure 13
Quartz resonator with series (load) capacitor

The relationship between f_s and f_L is given by

$$f_L \approx f_s \left(1 + \frac{C_1}{2(C_0 + C_L)} \right) \quad (4)$$

As shown in Figure 14, the distance to the parallel resonance is reduced in comparison to the series resonant quartz crystal (Figure 8). The reason is simple: The load resonant Quartz crystal has already been drawn in this direction according to equation 4 with the C_L of 15 pF. Shorted series capacitance C_L under 'Circuit' thus results again in the old distance (thin lines) from Figure 8, showing also the position of the 'series' resonant frequency.

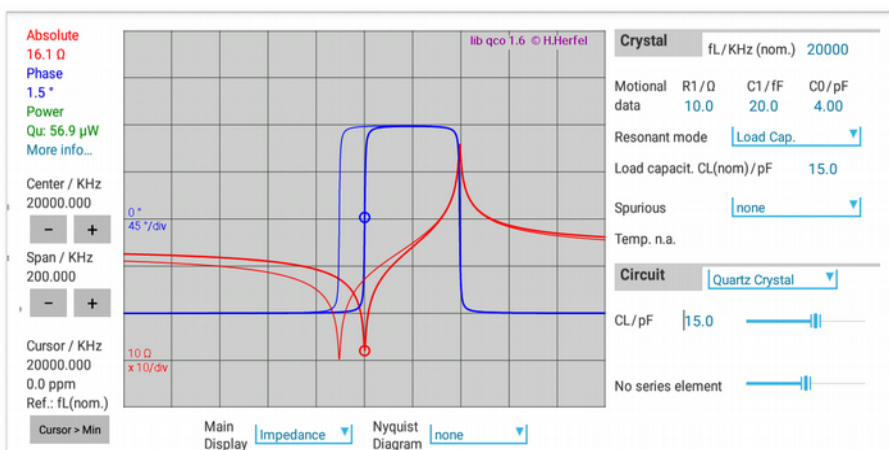


Figure 14
Quartz crystal with load capacitance
Position of load resonant frequency f_L and series resonant frequency f_s (thin lines)

Using load resonant resonators the user can apply a frequency correction in both directions by modifying the circuit load capacitance (pulling of the crystal). With a series resonant resonator and without additional inductances only an increase in frequency is possible.

By increasing the effective load capacitance in the circuit, the load resonant frequency f_L can be

reduced down to the actual series resonance frequency f_s . Conversely it can be increased up to the parallel resonance frequency f_p by decreasing the effective series capacitor. The possibility of change to either side not only allows to compensate for manufacturing tolerances, but is also the base for the specific frequency change in voltage-controlled oscillators. It should be noted that very low series (load) capacitances reduce stability drastically. In addition, the load resonant resistance increases to very high values.

This new total resistance of the series resonant circuit is the **load resonant resistance R_L**

$$R_L \approx R_1 \left(1 + \frac{C_0}{C_L}\right)^2 \quad (5)$$

In Figure 14, on the left, we see the load resonant resistance R_L as an 'Absolute' value of 16.1 Ω . This is also known as 'transformed R_r ' and is larger than the actual resonant resistance R_r of the quartz crystal without load capacitance. Without the circuit capacitance (circuit C_L set to infinity) we return to the non-transformed resonance resistance of 10 Ω . The value can be determined with better accuracy by reducing the span or by increasing the number of analyze points. A deviation can also, in particular for higher frequencies, be attributed to the effects of C_0 .

The relative difference between the resonant frequency and load resonant frequency is being called **load resonant frequency offset D_L**

$$D_L \approx \frac{C_1}{2(C_0 + C_L)} \quad (6)$$

The differential **pullability S** is

$$S = \frac{1}{f_s} \frac{\delta f_L}{\delta C_L} = -\frac{C_1}{2(C_0 + C_L)^2} \quad (7)$$

Summary:

Since a change with external adjustment capacitance is possible only in one direction (upwards) for a series resonant quartz crystal, whereas manufacturing inaccuracies usually go in both directions, the need arises to specify the nominal frequency together with a load capacitance for the manufacturing process. The series resonant frequency is then, as shown at the beginning of the chapter, a little lower and a manufacturing tolerance range as well as the limited accuracy of circuit values can be completely compensated for by circuit tuning.

Likewise, it should be noted that a frequency change downwards can also be achieved with a load inductance. This possibility should only be used when big tuning ranges are required: Inductors usually lack the adjustability, the accuracy and the temperature stability of capacitors.

In addition, it is clear that for voltage-controlled oscillators with large pulling ranges a rather big C_1 value is desirable. On the other hand high precision, high quality oscillators with high frequency stability should use resonators with small C_1 value.

1.4. Unwanted resonances of quartz crystals

In the selection box 'Spurious' an additional branch with motional parameters C_{1n} , L_{1n} and R_{1n} can be added in parallel to the quartz crystal with C_1 , L_1 and R_1 : a so-called spurious resonance. In Figure 15 it the option ' $R_{1n} = 10 \times R_1$ ' is selected; appropriate setting of the 'Temperature' slider results in a small peak beneath the main resonance f_r .

Incidentally, the parallel resonant frequency is now somewhat higher: In the simulation the C_{1n} of the spurious is assumed to be at 10% of the C_1 of the main resonance, thus contributing to the distance series / parallel resonance according to equation 2 (but only if the main and the disturbing series resonance are almost at the same frequency!).

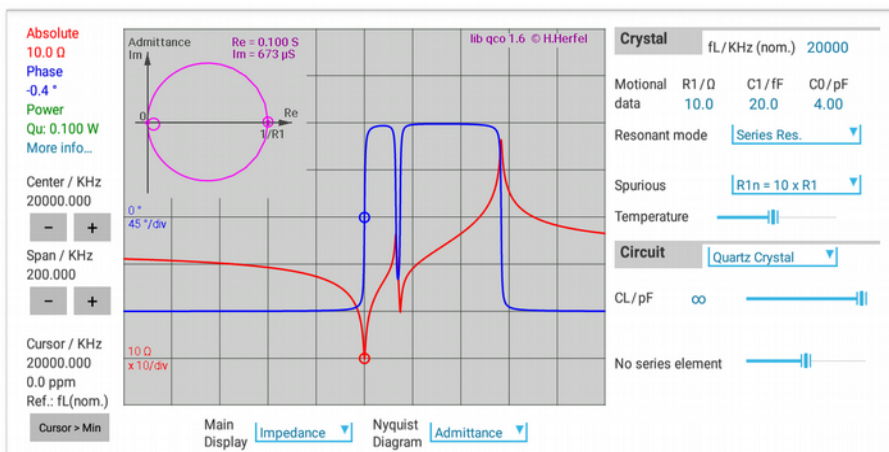


Figure 15
Series resonant quartz crystal with spurious resonance

These unwanted resonances usually show a big temperature dependence, in contrast to the stability of the desired vibration. You can change the 'temperature' in the simulation and thus the frequency of the spurious by moving the slider.

If a quartz crystal with load capacitance is used, the influence of spurious resonances can increase substantially (Figure 16). Depending on the value of the interfering parameters, the resonant resistance can significantly increase, the correct resonance can shift or even two equally strong resonance points can result.

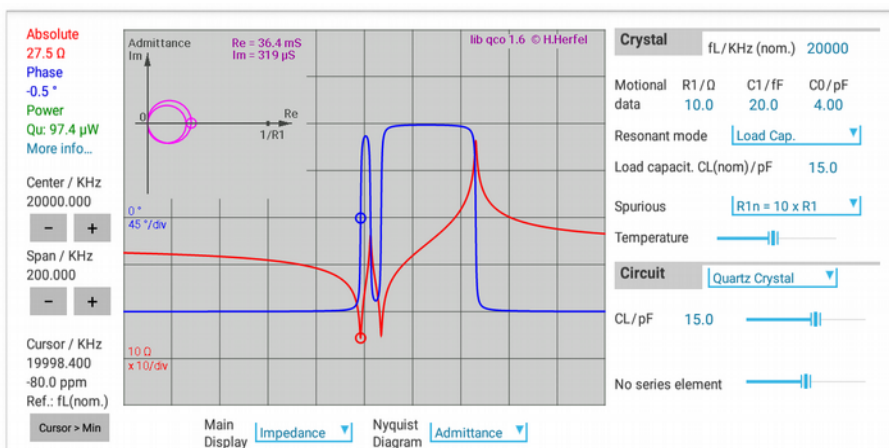


Figure 16
Quartz crystal with load capacitance
Stronger influence of a spurious resonance

2. Quartz Crystal Oscillators

2.1 Pierce oscillator with inverting amplifier

In most microprocessors a crystal oscillator in Pierce configuration is used as clock. The active part is taken by an inverter.

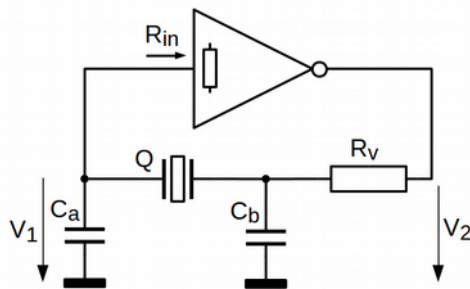


Figure 17

Pierce oscillator with inverter and feedback network for resonators in fundamental mode (Elements of the supply or for the stabilization of the operating point are not shown)

The output resistance of the inverter together with the resistor R_v and the PI-Element C_b / Resonator / C_a constitute a narrow band pass filter with frequency-dependent phase shift. The oscillation condition requires the total phase to be 360° . The inverter provides 180° , usually with a bit more because of additional semiconductor-related time delays. The external phase shift by the bandpass may therefore be slightly less than 180° .

Choose the values of the external components so that both the amplitude and the phase component of the resonant condition is satisfied without exceeding the maximum crystal load; usually, a good compromise can be found. Choosing the same value for C_a and C_b , e.g. the double value of C_L , does not make sense. The better solution is an impedance transformation by an 'unbalanced' configuration of the capacitors connected to the resonator. A first guess for C_a is 10 - 20% higher than the specified load capacitance, a first value for C_b is at least twice the C_a value.

R_v should be chosen big enough that the allowed level of the crystal load will not be exceeded. With the sliders for R_v and C_b the transfer function should be optimized for the right resonator load and an optimum shape of the resonance curve. This setting is not necessarily the one with the highest voltage gain of the feedback network, given the inverter provides sufficient gain!

Within the menu 'Structure' it is possible to select and simulate the feedback network of such a Pierce oscillator.

In Figure 18 (next page) the advantages of a matched configuration with C_a , C_b , and R_v can be clearly seen: The resonance and the phase behaviour are much better, maximum amplitude and best phase of 180° coincide.

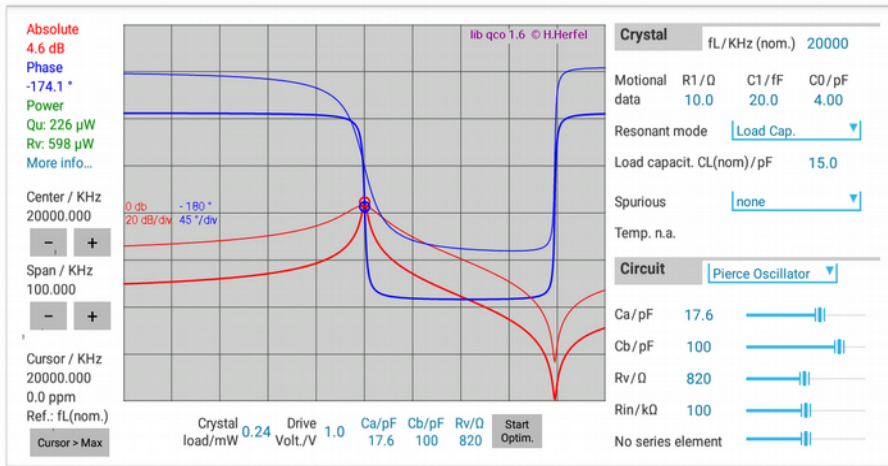


Figure 18

Pierce oscillator, symmetrical (22pF / 22pF / 300Ω) and asymmetrical choice of C_a and C_b (18 / 110pF / 820Ω)

Figure 19 shows again the unfavourable influence of a too-low resistance R_v . The permissible level of the resonator load is exceeded. Furthermore, the necessary phase shift of 180° is only just reached at a point with low phase slope. Moreover, maximum amplitude and best phase of 180° do not coincide at the same frequency.

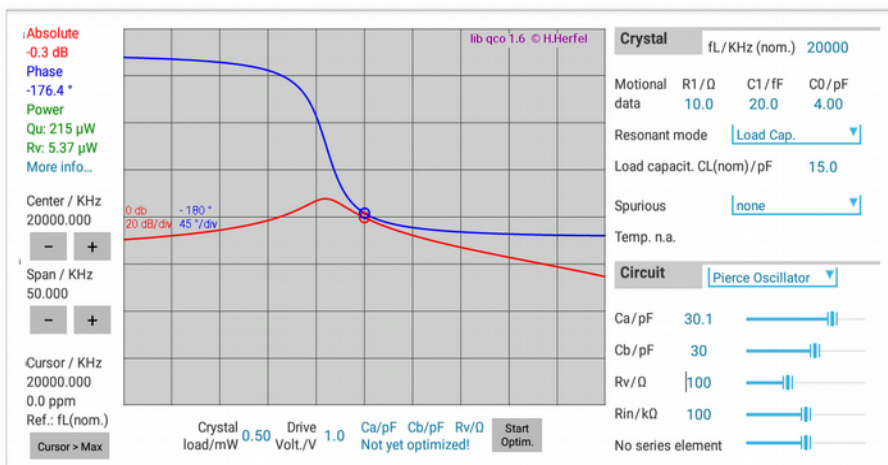


Figure 19

Pierce oscillator with a too low R_v

The input impedance of the inverter can be simulated using the slider for R_{in} .

In the final circuit C_a and R_b should be reduced by the values of the input capacitance and the output resistance of the inverter.

The Pierce oscillator requires a quartz crystal with load capacitance, when no additional inductors are connected in series to the resonator.

A possible circuit for overtone resonators is shown in Figure 20 (next page).

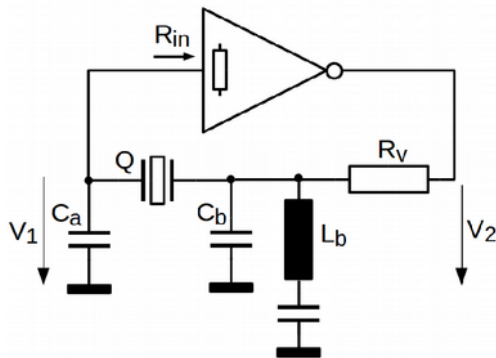


Figure 20

Pierce oscillator with inverter and feedback network for resonators in overtone mode
(Elements of the supply or for the stabilization of the operating point are not shown)

2.2 Colpitts oscillator with non-inverting amplifier

For the Colpitts oscillator a non-inverting element supplies the necessary gain. The menu 'Structure' of the app offers the possibility to simulate the feedback network of this oscillator type.

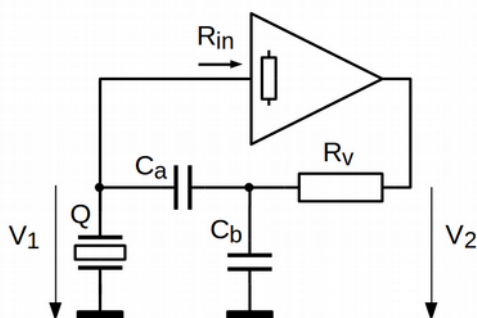


Figure 21

Colpitts oscillator with non-inverting amplifier and feedback network for fundamental mode resonators
(Elements of the supply or for the stabilization of the operating point are not shown)

In this structure it is immediately evident that the loop of C_b , C_a and resonator configure a resonant circuit (a lossy one due to the R_1 of the resonator). The resonator takes the role of the inductance. The amplifier provides via R_v for the maintenance of the oscillation in the loop.

Equivalently the situation can be analyzed with a two-port consideration as in the case of the Pierce oscillator.

For this circuit too, it is recommended to use an impedance transformation by selecting an 'unbalanced' choice of capacitance values (Figure 22, next page).

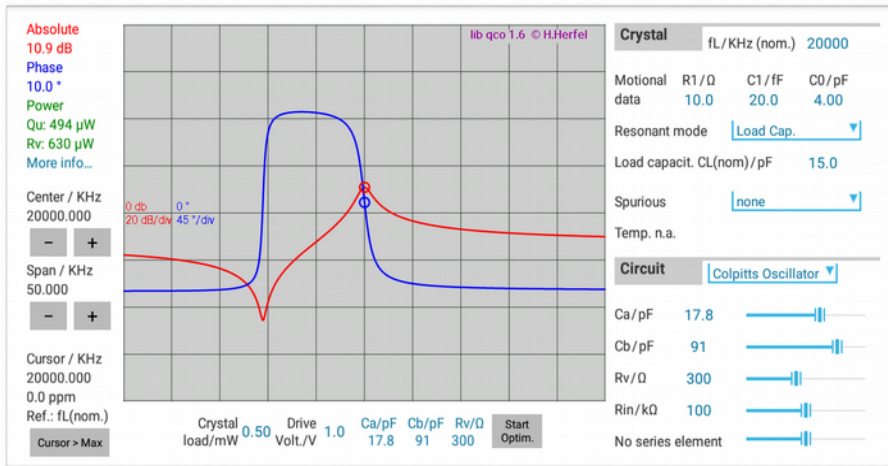


Figure 22
Colpitts-Oscillator
Transfer function of the feedback network

2.3 Colpitts oscillator with bipolar transistor

When a Colpitts oscillator is configured in common-collector mode (emitter follower), the output resistance r_a of the transistor stage at the emitter (or the source for FETs) can replace the value of R_v .

For a bipolar transistors this output resistance is

$$r_a = \left(\frac{1}{S} + \frac{R_g}{\beta} \right) \parallel R_E \quad (8)$$

$$r_a = \left(\frac{26mV}{I_c} + \frac{R_g}{\beta} \right) \parallel R_E \quad (8a)$$

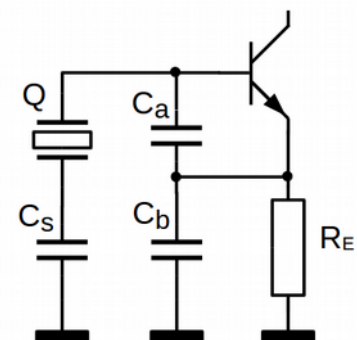


Bild 23

Colpitts-oscillator with bipolar transistor and additional series capacitance C_s

(Elements of the supply or for the stabilization of the operating point are not shown)

Calculation example:

$$R_E = 2 \text{ k}\Omega$$

$$I_c = 0,5 \text{ mA}$$

$$R_g = 100 \text{ k}\Omega (= R_{in}, \text{ i.e. the voltage divider at the base})$$

$$\beta = 200$$

$$r_a = (52 \Omega + 500 \Omega) \parallel 2 \text{ k}\Omega = 432 \Omega$$

Hence: there are some degrees of freedom which allow to set the values of the circuit in accordance with the optimal R_v .

In the shown example a capacitance C_s is put in series to the crystal. It may serve as a frequency tuning element; additionally the capacitances C_a and C_b have a higher value. This helps to reduce the influence of temperature dependent transistor parameters.

The app allows to select this capacitance C_s within a reasonable value range. Figure 24 shows an optimization example.

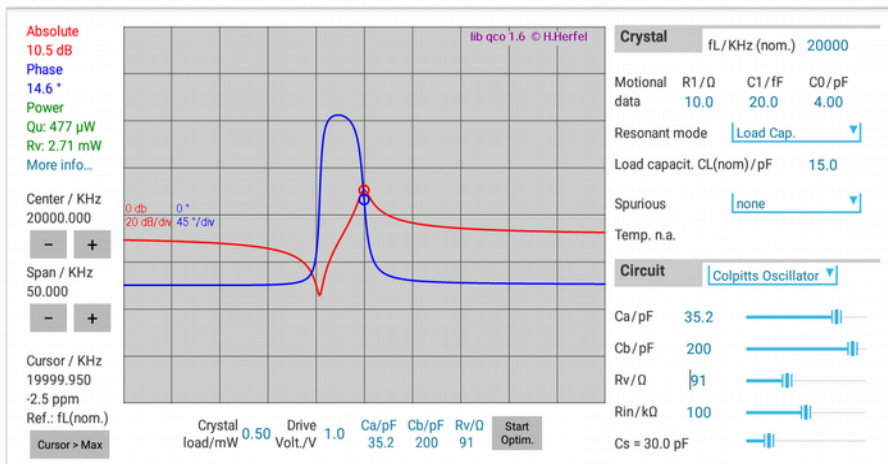


Figure 24

Colpitts oscillator with bipolar transistor and additional series capacitance at the crystal
Transfer function of the optimized feedback network with an additional series capacitance at the crystal