Direct-Mod Analog Link Noise

A directly-modulated fiber optic link consists of a fiber-coupled laser diode, a length of fiber with perhaps some optical attenuation, and a photoreceiver. The laser diode is DC-biased at a current above threshold, and the modulating RF signal is coupled directly onto the diode bias. The photoreceiver is assumed in this analysis to be a PIN diode.

When we speak of link noise, we are typically concerned with the amount of undesired spectral noise that is *absorbed by the load,* or the *output* of the link. This output noise determines the lowest level signal that can be resolved at the output (if there is no microwave link, there is no microwave noise). There are 3 sources to this output noise: thermal, shot and RIN.

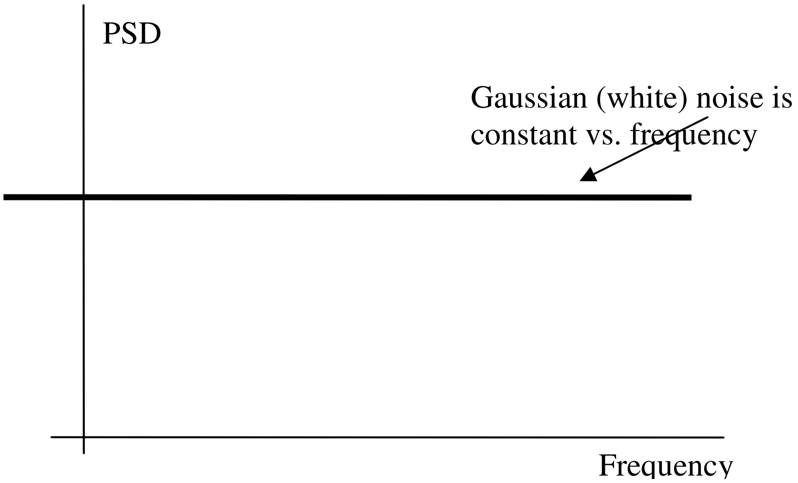
Thermal Noise. This is the ubiquitous noise present in all RF systems, often referred to as Johnson noise. For an understanding of thermal noise, its mathematical treatment, and the definitions of noise figure, review Goldberg's landmark paper, circa 1930's. In a F/O link, thermal noise at the input terminals is transferred to the output load by the gain of the system. Every real resistor in the link chain also generates thermal noise, i.e. the laser resistance generates thermal noise proportional to the mean-square current through it, and that noise is transferred to the output by the gain or loss from that point forward. Similarly, the PIN diode real resistance generates noise, as do any real resistances in the microwave output circuitry, and the load resistor itself.

This output noise is *spectrally flat*. This is an important concept, and it implies that the random process that determines the amplitude of the noise voltage is *Gaussian*, also referred to as *white noise*. If you have a signal with random amplitude, you cannot predict its amplitude at any time. However, from random process theory, it is known that the Power Spectral Density (PSD) of the random signal is equal to the Fourier Transform of its Autocorrelation function. So even though you can't predict the amplitude of the signal, if you know the signal's statistical properties, you can determine how much power it signal carries.

Keep in mind we're talking about PSD here, not absolute power. The amount of power absorbed by the load is the integral of the PSD over the desired bandwidth. In the case of white noise that is a simple integration process, linearly dependent on the bandwidth, because the PSD is flat.

If we're building a link that carries FM video, for example, then at some point after the photoreceiver there will be a filter that will pluck off a video channel. FM video has a channel bandwidth of 36 MHz; this is the integration bandwidth. Multiplying the output white noise density by 36 MHz gives the noise power.

Because we, as the link makers, don't always know what the application bandwidth of our users will be, we often refer the noise to a bandwidth of 1 Hz. As long as the noise is Gaussian, this is acceptable.



The calculation and understanding of all the thermal noise sources in a F/O link can usually be reduced to one simple conceptual point: the link is lossy. Since the link is lossy, all of the thermal noises generated along the link (in the laser, in the diode, etc) are attenuated before they reach the load. The only measurable output thermal noise is therefore *from the load itself*. In a link at ambient temperature, this noise level is:

$$Nth = kTB \tag{1}$$

where T is the temeprarure (K), B is the bandwidth (Hz), and k is Boltzman's constant:

$$k = 1.381(10)^{-23} J/K\,. \tag{2}$$

Two things to note here: 1) the output noise power is independent of the load resistor. It's still the same noise level, whether a 1 ohm system or a 1 megohm system, and 2) k relates energy and temperature. This comes from thermodynamics. Random molecular motion is linearly dependent on how hot something is. The molecules bounce around and bump into each other, and *dissipate kinetic energy in the form of heat*. The energy is spectrally flat from DC, through microwave frequencies, and into the UV range. The relation that says that the energy is linearly dependent on the temperature is valid for all temperatures that concern we humans. It falls apart at the low extreme (very close to absolute zero) and the high extreme (when the particles become a plasma, such as on the surface of the sun or in a nuclear explosion). At very low temperatures interesting things happen that lead to no energy dissipation, also known as superconductivity. These don't concern us.

So the thermal noise has units of (J)(Hz) = J/sec = Watts (Power). If we normalize to a 1-Hz band for reasons described above, we leave out the B and we end up with units of W/Hz (Power Density).

At ambient temperature (293K) we get Nth = $4.05(10)^{-21}$ W/Hz, or $4.05(10)^{-18}$ mW/Hz, or –173.9 dBm/Hz. So we usually say that the output noise is –174 dBm/Hz at ambient temp. Remember the limitations of this statement:

1) The link is lossy, i.e. *there are no gain elements anywhere in the link.* This is a broad assumption. If it's not true, then each thermal noise contributor from every real resistance in the system must be evaluated separately; a difficult process. If there are preamps or postamps, then it is often easier to evaluate the link without amplifiers first, then figure out the impact of adding the amps. If there is gain embedded in the link, such as an EDFA or other optical amp, it will generate undesired optical signals due to Amplified Spontaneous Emission (ASE). This noise also gets demodulated by the photoreceiver, and appears at the output load. We generally treat the added noise from the amp as a fourth noise component at the output. This paper does not address those cases, but it should be noted that the ASE noise from an amplifier could be the dominant microwave noise component in a fiber link.
2) The temperature is ambient (25C). If you are at a different temperature, then you have to modify the thermal component accordingly.

Shot Noise. Light that impinges on the intrinsic region of the P-I-N diode causes electron-hole pairs to recombine. The recombination process is discrete or quantized; each electron contributes a charge of exactly q coulombs to the total flow of charge. Charge "packets" enter the circuit randomly, in the same way that buckshot would strike a target. Hence the term "shot" noise, coined by Schottky in 1918 in reference to electron combinations at the plate of a vacuum tube. Shot noise occurs in tubes and PN junctions.

Each "shot" or charge packet appears as a delta function in the time domain, with randomly-spaced intervals that follow a Poisson probability distribution. Over time, the result of these charge packets is a current with an average value (DC photocurrent), and a noise current that is spectrally flat and whose mean-square value is given by

$$< i >^2 = 2qIB \qquad (3)$$

where q is the electron charge ($1.602(10)^{-19}$ coulombs) and I is the average photocurrent, equal to the product of the diode responsivity and the average optical power. It should be intuitive that the noise current is spectrally flat, or *white*, by recalling that the Fourier transform of a delta function is constant for all frequency.

The mean-square noise current generates shot noise power in the load,

$$Nshot = 2qIBz_0 \qquad (4)$$

This white noise extends from DC to the quantum threshold frequency of the diode – well past the microwave region. The total noise is that which is integrated by the microwave circuit bandwidth. Also note the units of N_{shot}: (C)(A)(Hz)(Ω) = (A)(sec)(A)(Hz)(Ω) = $A^2\Omega$ = Watts. Again, in a 1-Hz band, we can define this as W/Hz.

We can simplify using logarithms. Assuming a 50Ω load, the shot noise is

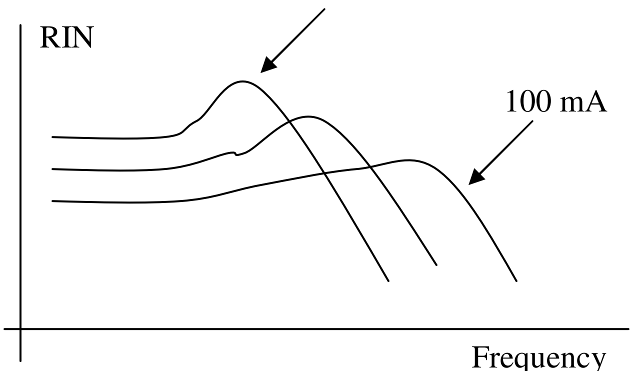
$$Nshot = 10\log(2qz_0) + 10\log(I) = -168 + 10\log(I) \quad \text{dBm/Hz} \tag{5}$$

where I is in mA.

Laser Relative Intensity Noise (RIN). A laser generates an optical carrier by the process of exciting electrons to a defined energy value, then stimulating their precipitation to a lower energy level of a defined energy transition. If all of the electrons follow the same transition, then all of the generated photons will have the same energy (wavelength). The result will be a perfectly correlated delta function optical carrier. In practice, thermal and other considerations (including shot noise within the laser junction!) cause transitions to undesired states, yielding photons that are not at the same frequency or phase as the carrier. This appears as optical noise. The noise is transferred to the photoreceiver, demodulated and delivered to the microwave load as a noise power.

RIN noise is not white. It has microwave properties that extend from the optical carrier out to the relaxation oscillation frequency of the laser diode. The RIN frequency shape is dependent on the laser bias current. At low bias, the RIN peaks near the relaxation oscillation frequency of the laser diode. At high bias current, the peaking diminishes and the noise density becomes flatter.

The photodetection process translates the optical carrier frequency to DC, so the microwave RIN noise falls directly into the microwave modulation bandwidth of the link. Although RIN is not white noise, it can usually be treated as white noise in a relatively narrow band around the frequency of interest.



RIN is the result of a series of noise-generating elements within the laser structure, and no closed-form expression has been derived for it. It is statistically defined as the ratio of the spectral density of the square of the optical power fluctuations to the square of the average optical power:

$$RIN(f) = \frac{< \Delta P^2(f) >}{P_L^2} \tag{6}$$

RIN is defined as a ratio of a squared frequency-dependent power to a squared average power; it has odd units of W^2/Hz per W^2, which is simply time, or dB/Hz in logarithmic terms. Typical distributed feedback lasers used for direct modulation links will exhibit RIN noise on the order of 10^{-15}/Hz or –150 dB/Hz. This noise power is propagated along with the optical carrier, and both are amplified or attenuated by the same amount, so that the detected RIN at the receiver has a fixed relationship to the average optical power. The average optical power incident on the photoreceiver generates a RIN-noise photocurrent that is delivered to the load as a noise power density:

$$N_{rin}(f) = I^2 \cdot RIN(f) \cdot z_0 \quad \text{W/Hz} \tag{7}$$

where I is the DC photocurrent. If we assume a microwave detection bandwidth where the RIN level is flat, then we can use a white noise approximation:

$$N_{rin} = I^2 \cdot RIN \cdot z_0 \cdot B \quad \text{W} \tag{8}$$

Taking the logarithm in a 1-Hz band with a 50Ω load:

$$N_{rin} = -13 + 20\log(I) + RIN \quad \text{dBm/Hz} \tag{9}$$

where I is in mA and RIN is in dB/Hz. The –13 term comes from converting from W to mW by subtracting 30 dB from 10log(50).

Link Output Noise. Putting at all together, the 3 noise components described here are derived from independent sources within the link. Therefore their power levels add arithmetically at the output. For a lossy link which is directly terminated into a 50Ω load, there are only two variable quantities: RIN and photocurrent.

The thermal noise component is independent of photocurrent. The shot noise component varies 1:1 with photocurrent, and the RIN noise component varies twice as fast as the photocurrent. This implies that, at low photocurrent (low optical drive into the receiver) thermal noise will dominate the output. At high drive, RIN noise dominates. There may also be a region in which the shot noise is dominant, at moderate optical drive. Here we assume that the optical power (and photocurrent) may change because of changes in optical attenuation, such as longer fiber runs or addition of connectors, and not because of changes to the laser output power level.

Contrast the noise output with the RF gain of the link. If the optical power (photocurrent) increases, the RF gain increases twice as fast. At low photocurrent, when the system is thermal noise limited, the RF gain increases with optical power, but the noise level does not. This implies that the SNR increases 2 dB with every dB increase in photocurrent. In a shot noise limited region, the SNR continues to increase 1 dB for every dB of photocurrent. In a RIN noise limited system, the SNR is as high as it can get, and does not change with photocurrent, because both the RF gain and the noise level are now increasing 2 dB for every dB of photocurrent. These concepts are highlighted in the figure on the following page, for RIN = -155 dB/Hz.

Finding RIN and Noise Figure. Noise figure is defined for thermal-limited input, in which case the output noise power is given by:

$$N_{out} = kT_o BGF \tag{10}$$

where G is the link transducer gain and T_o is room temperature (293K). In this case, noise figure is found directly from the output noise density as

$$F(dB) = N_{out}(dBm/Hz) + 174 - G(dB) \tag{11}$$

Often we want to find the value of the laser RIN at a particular frequency and laser bias. In this case, we can subtract the thermal and shot components from the output noise, leaving only the RIN component. It is important that the output noise measurement be made in a RIN limited region; otherwise the resulting arithmetic will be error-prone due to the subtraction of large numbers.

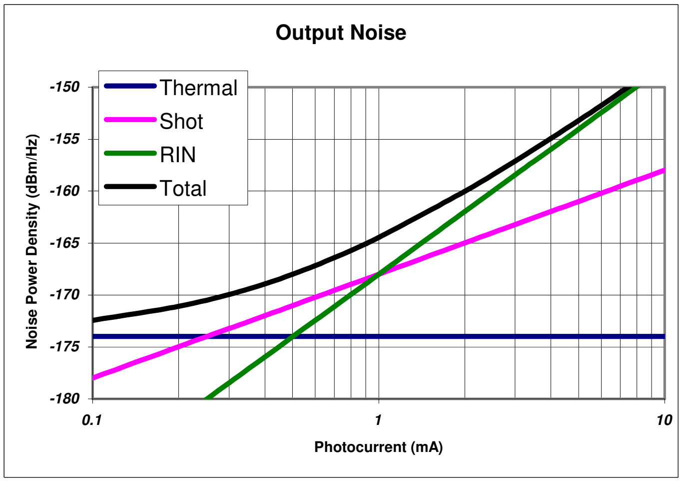
Example. The output noise power of a lossy direct-mod link at 3 GHz is measured and found to be equal to –153.2 dBm/Hz. The test receiver is directly terminated in a 50Ω load, and has photocurrent of 5 mA when the noise is measured. We can find and eliminate the contributions from thermal and shot noise as follows:

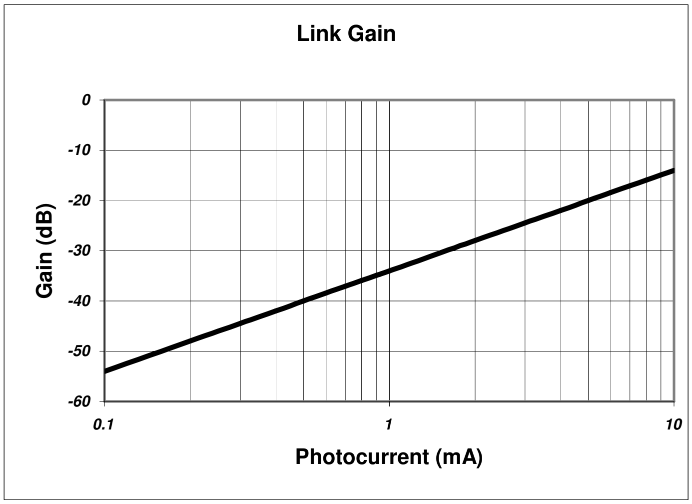
$$N_{shot} = -168 + 10\log(5) = -161 \text{ dBm/Hz} \tag{12}$$

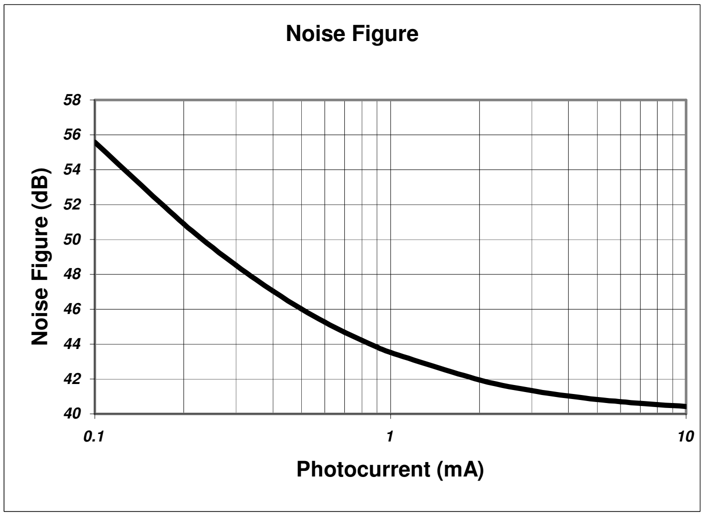
$$N_{rin} = N_{out} - N_{shot} - N_{th} = 10\log\left[10^{\frac{-153.2}{10}} - 10^{\frac{-161}{10}} - 10^{\frac{-174}{10}}\right] = -154 \text{ dBm/Hz} \tag{13}$$

from which the laser RIN is found:

$$RIN = N_{rin} + 13 - 20\log(5) = -155 \text{ dB/Hz} \tag{14}$$


Output Noise
Thermal
Shot
RIN
Total
Noise Power Density (dBm/Hz)
-150
-155
-160
-165
-170
-175
-180
0.1
1
10
Photocurrent (mA)


Link Gain
Gain (dB)
0
-10
-20
-30
-40
-50
-60
0.1
1
10
Photocurrent (mA)


Noise Figure
Noise Figure (dB)
58
56
54
52
50
48
46
44
42
40
0.1
1
10
Photocurrent (mA)

Further Considerations. When performing microwave measurements on a direct mod link there are considerations that must be addressed as part of system calibration. These include:

Photoreceiver not directly terminated into 50Ω.
If the photoreceiver has microwave loss between the diode and the load, the shot and RIN noise levels will be reduced by the loss, but the thermal component will remain the same at the output.

Microwave Amplifiers.
Amplifiers at the input and/or output will affect the noise levels. The test engineer must correct for these via the appropriate cascade gain and noise analysis.

Output noise level is very low.
An unamplified link will generally have output noise level below the measurement capability of the test equipment. The test engineer will need to use a post amplifier with known gain and noise figure, and de-embed the link output noise from the cascaded measurement.