System Identification Approach
applied to
Jitter Estimation.

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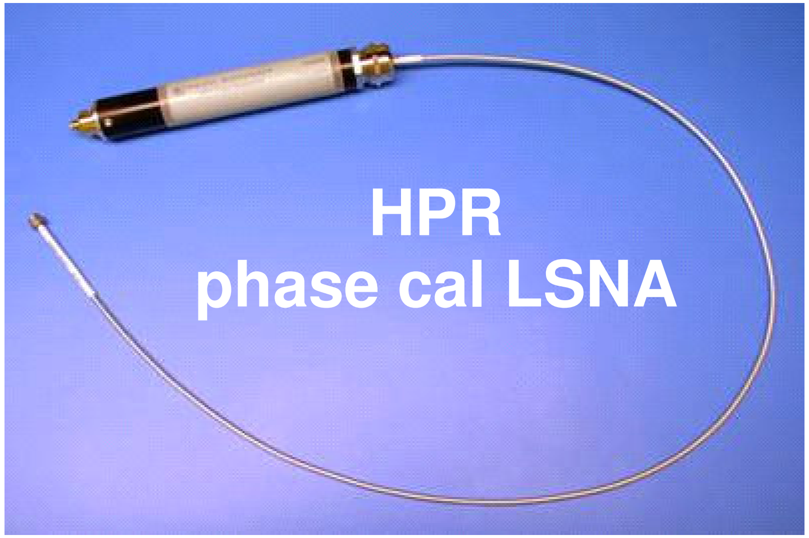
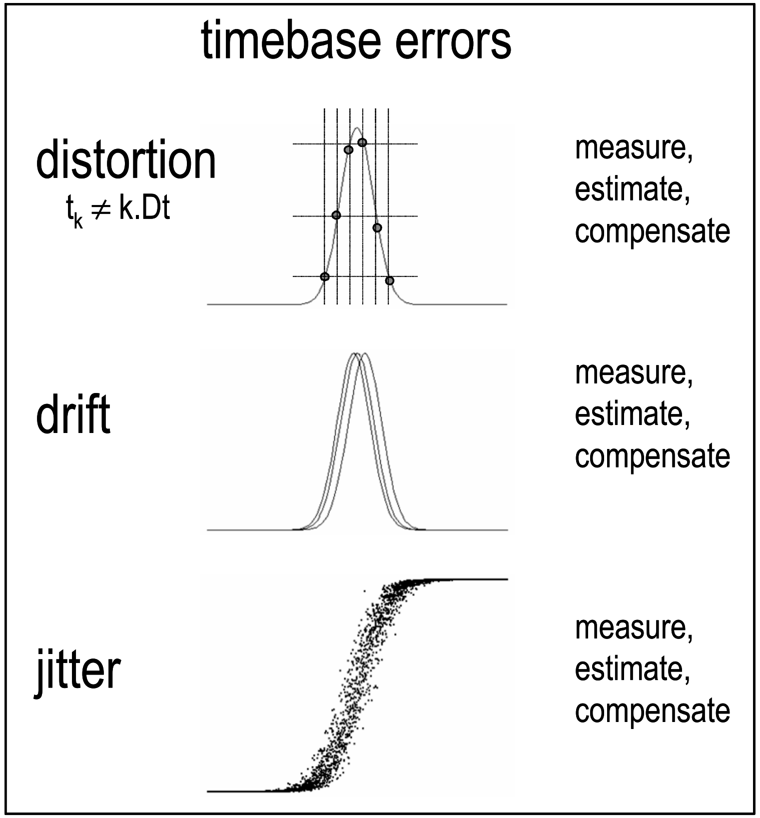
Motivation



calibrated sampling oscilloscope



Model

$$y(t_i) = y_0(t_i + n_t(t_i)) + n_y(t_i)$$

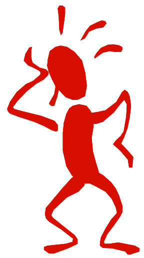
$$n_t \sim N(0, \sigma_{n_t}) \quad n_y \sim N(0, \sigma_{n_y})$$

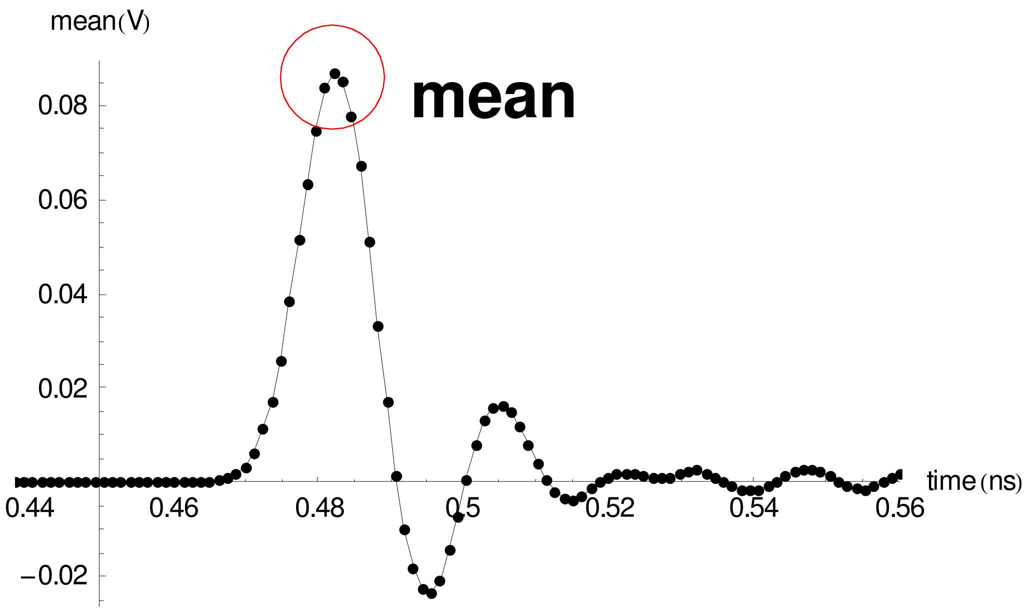
1st order model

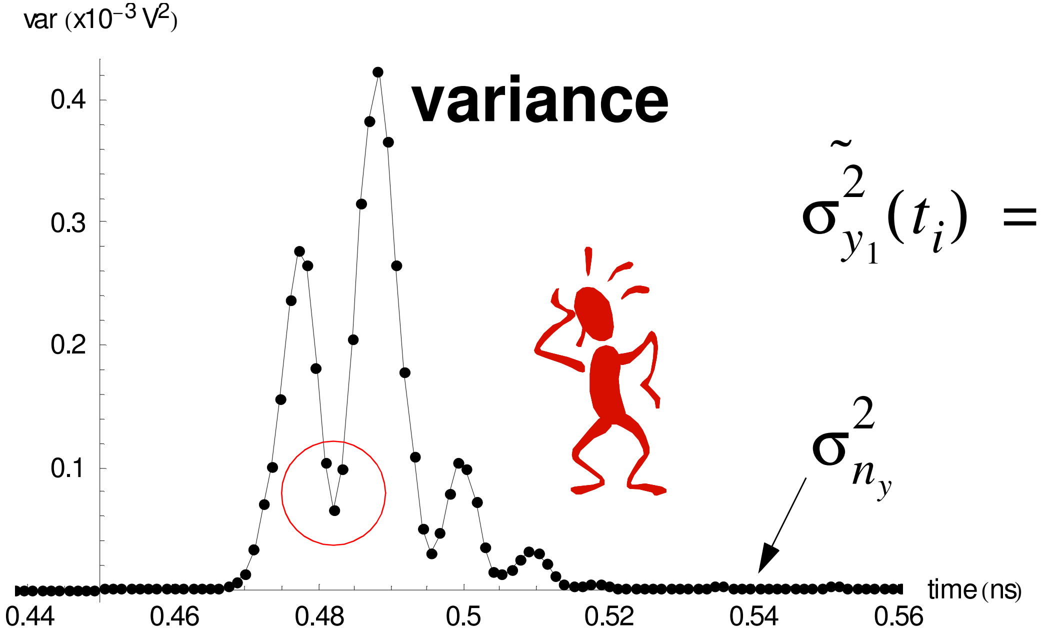
$$\tilde{y}_1(t_i) = y_0(t_i) + \left.\frac{dy_0}{dt}\right|_{t = t_i} \cdot n_t(t_i) + n_y(t_i)$$

$$E[\tilde{y}_1(t_i)] = y_0(t_i)$$

$$\sigma^2_{\tilde{y}_1}(t_i) = \sigma^2_{n_y} + \left.\left(\frac{dy_0}{dt}\right)^2\right|_{t = t_i} \cdot \sigma^2_{n_t}$$







$$\tilde{\sigma}_{y_1}^2(t_i) = \sigma_{n_y}^2 + \left(\frac{dy_0}{dt}\right)^2\Bigg|_{t=t_i} \cdot \sigma_{n_t}^2$$

Extended model

3rd order model

$$\tilde{y}_3(t_i) = y_0(t_i) + \sum_{k=1}^{3} \frac{1}{k!} \cdot d^k y_0 / dt^k \Big|_{t = t_i} \cdot n_t^k(t_i) + n_y(t_i)$$

$$E[\tilde{y}_3(t_i)] = y_0(t_i) + \frac{1}{2} \cdot (d^2 y_0 / dt^2)\Big|_{t = t_i} \cdot \sigma_{n_t}^2$$

$$y_0(t) = A \sin \omega t \quad \Rightarrow$$

$$E[\tilde{y}_3(t_i)] = A(1 - \omega^2 \sigma_{n_t}^2 / 2) \cdot \sin \omega t$$

$$\begin{aligned}\tilde{\sigma}_{y_3}^2(t_i) = {} & \sigma_{n_y}^2 + (dy_0/dt)^2\Big|_{t = t_i} \cdot \sigma_{n_t}^2 \\ & + [1/2 \cdot (d^2 y_0/dt^2)^2 + (dy_0/dt) \cdot (d^3 y_0/dt^3)]\Big|_{t = t_i} \cdot \sigma_{n_t}^4 \\ & + 5/12 \cdot (d^3 y_0/dt^3)^2\Big|_{t = t_i} \cdot \sigma_{n_t}^6\end{aligned}$$

Estimators

$$V_{(W)LS} = \sum_{i=1}^{N} e^2(t_i) \qquad e(t_i) = \left[\sigma_{t_i}^2 - \tilde{\sigma}_y^2(t_i, \theta)\right] / W_i$$

model

$$\theta = [\sigma_{n_y}^2, \sigma_{n_t}^2]^T$$

estimates

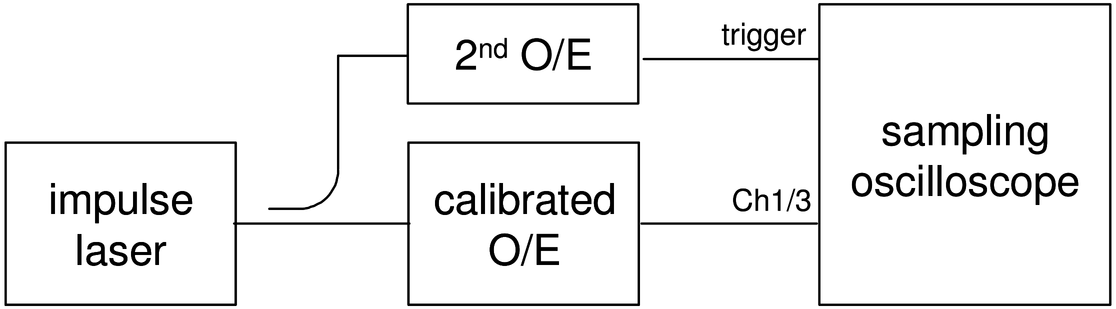
$$X_i \sim N(0, 1) \qquad \Rightarrow \qquad \sum_{i=1}^{r} X_i^2 \sim \chi_r^2$$

$$V_{ML} = n/2 \cdot \sum_{i=1}^{N} \left[\ln \tilde{\sigma}_y^2(t_i, \theta) + \sigma_{t_i}^2 / \tilde{\sigma}_y^2(t_i, \theta)\right]$$

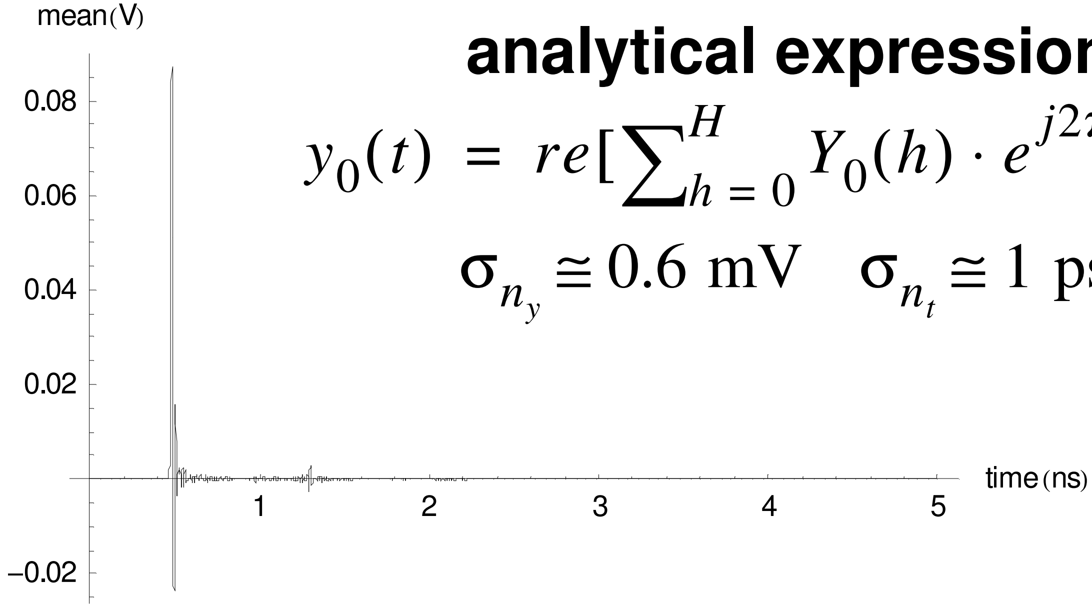
solve nonlinear problem

Simulations

"as realistic as possible"



Tracy Clement @ NIST



analytical expression:

$$y_0(t) = re[\sum_{h=0}^{H} Y_0(h) \cdot e^{j2\pi h \cdot \Delta f \cdot t}]$$

$$\sigma_{n_y} \cong 0.6 \text{ mV} \quad \sigma_{n_t} \cong 1 \text{ ps}$$

Simulations

$$\sigma_{n_y} = 0.6 \text{ mV} \quad \sigma_{n_t} = 0 \text{ .. } 2 \text{ ps} \ (\Delta = 0.2 \text{ ps})$$

$$y_0(t) = re\left[\sum_{h=0}^{H} Y_0(h) \cdot e^{j2\pi h \cdot \Delta f \cdot t}\right]$$

step 1: use 3rd order model to generate data (variance)

$$\tilde{\sigma}^2_{y_3}(t_i) = \sigma^2_{n_y} + (dy_0/dt)^2\Big|_{t=t_i} \cdot \sigma^2_{n_t} + [1/2 \cdot (d^2y_0/dt^2)^2 + (dy_0/dt) \cdot (d^3y_0/dt^3)]\Big|_{t=t_i} \cdot \sigma^4_{n_t} + 5/12 \cdot (d^3y_0/dt^3)^2\Big|_{t=t_i} \cdot \sigma^6_{n_t}$$

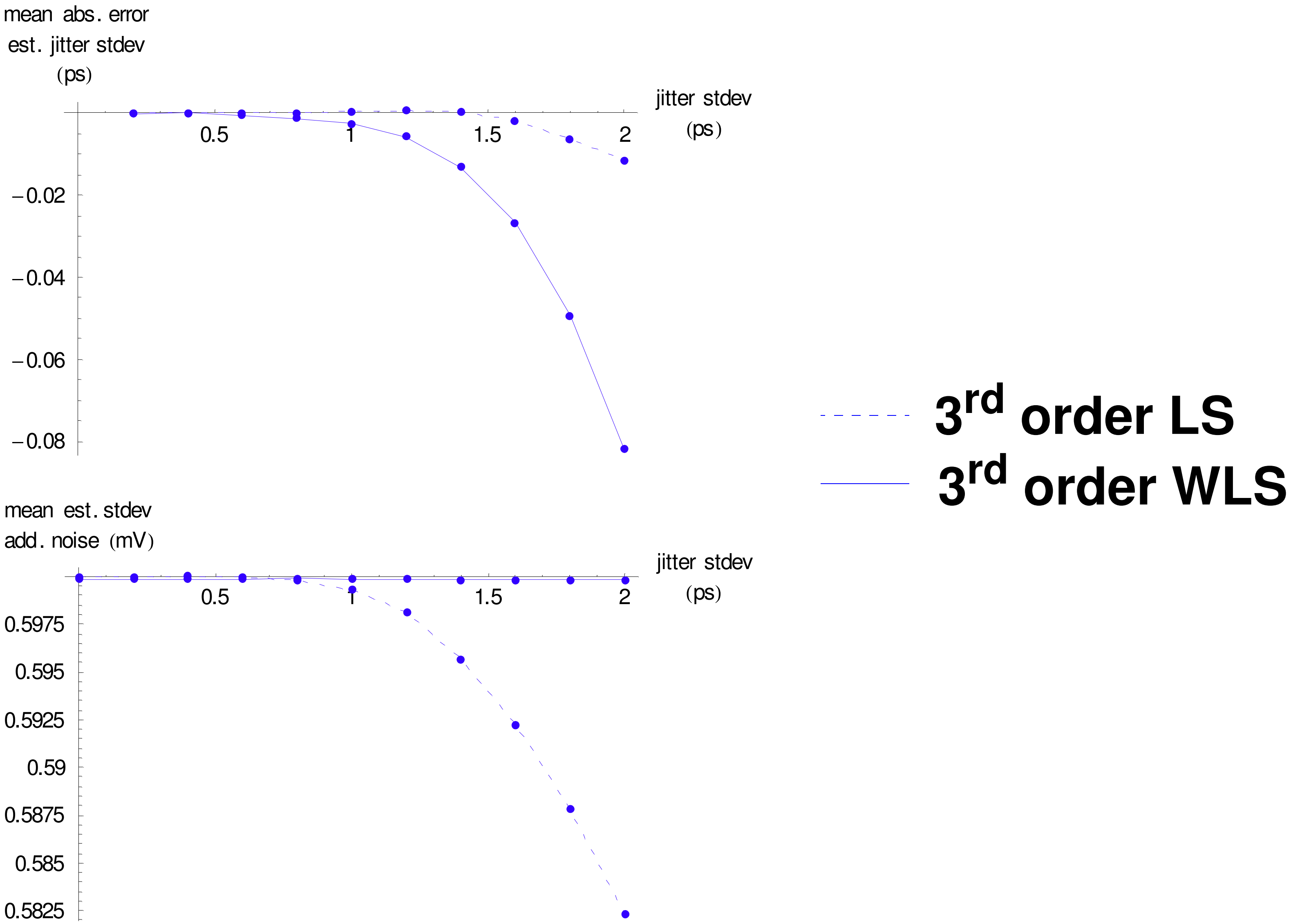
step 2 & 3: use realistic variance

$$y(t_i) = y_0(t_i + n_t(t_i)) + n_y(t_i)$$

Simulation results step 3: use realistic variance

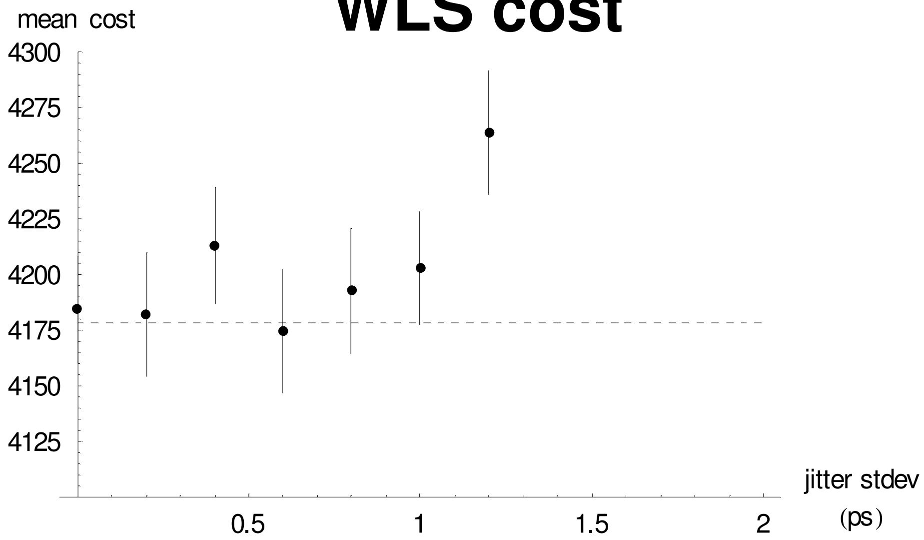
jitter

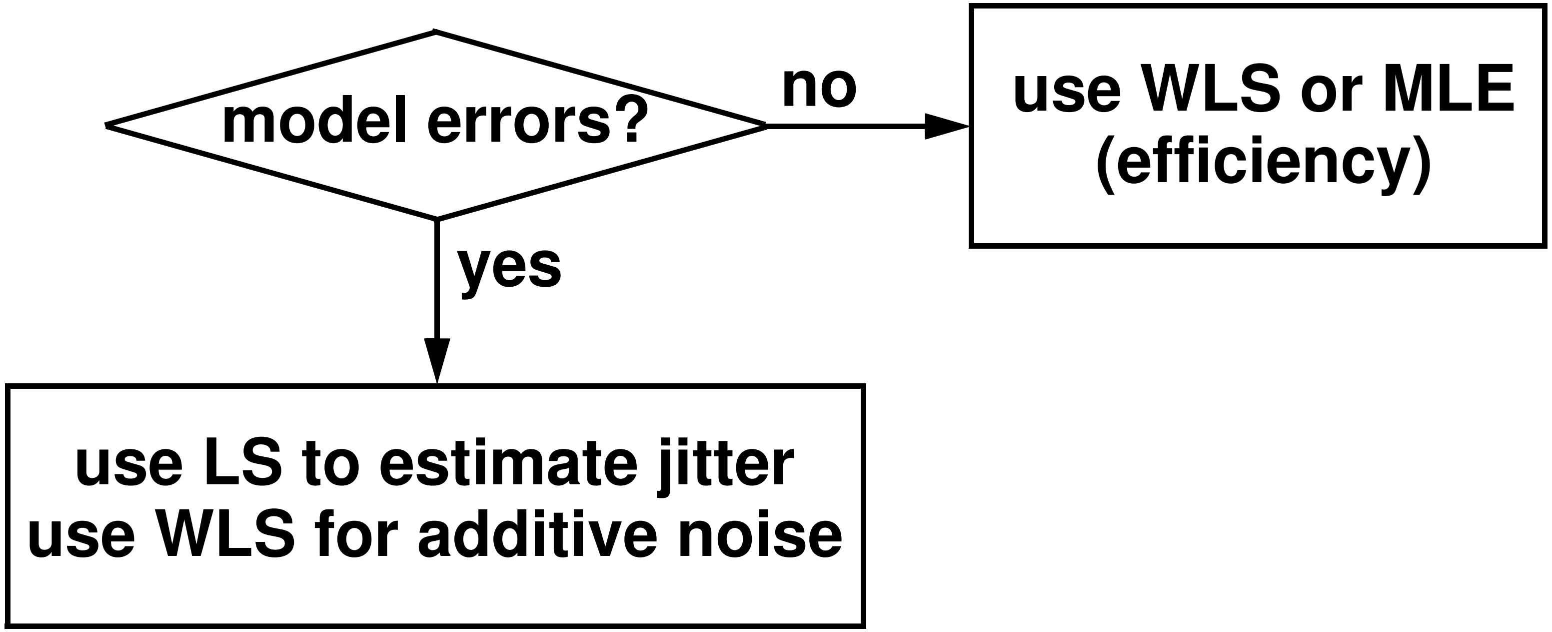
additive



Guidelines

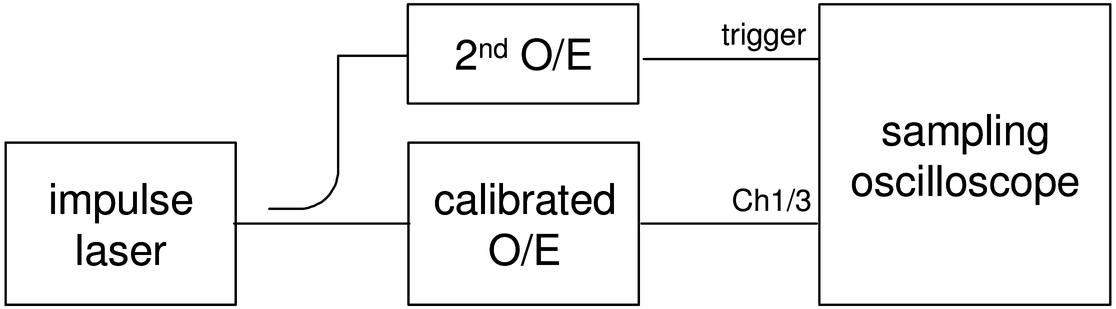
WLS cost



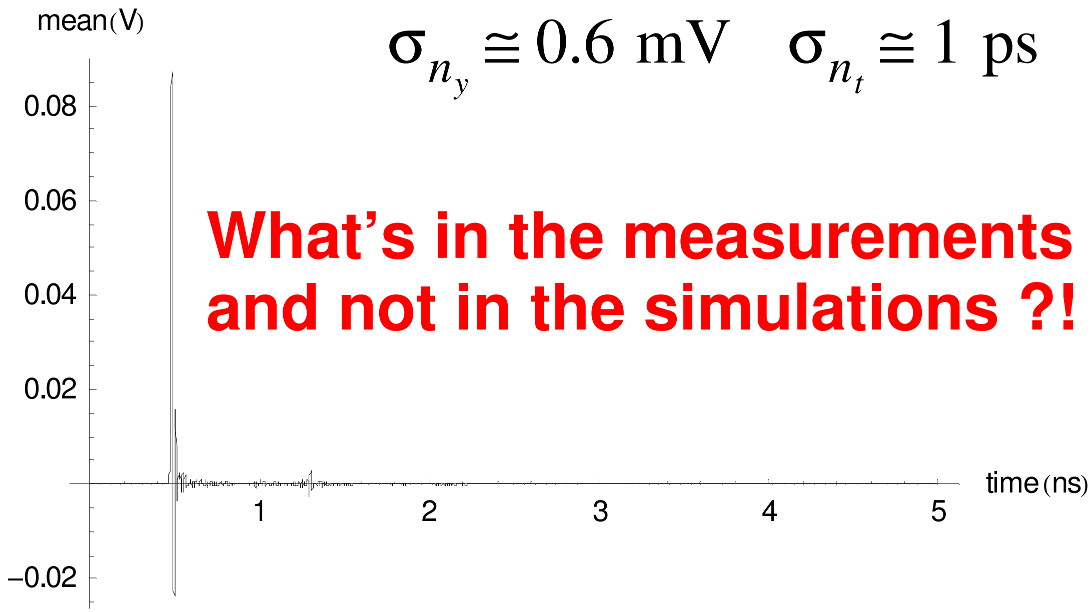


Measurements

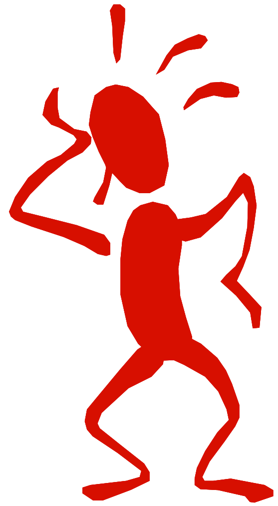
poor fit of variance



$$\sigma_{n_y} \cong 0.6\ \mathrm{mV} \quad \sigma_{n_t} \cong 1\ \mathrm{ps}$$



What's in the measurements and not in the simulations ?!



Time base drift

given arbitrary τ

variance of delayed signal $\neq$ delayed variance of signal

except for $\tau = m\Delta t \quad (m \in Z)$

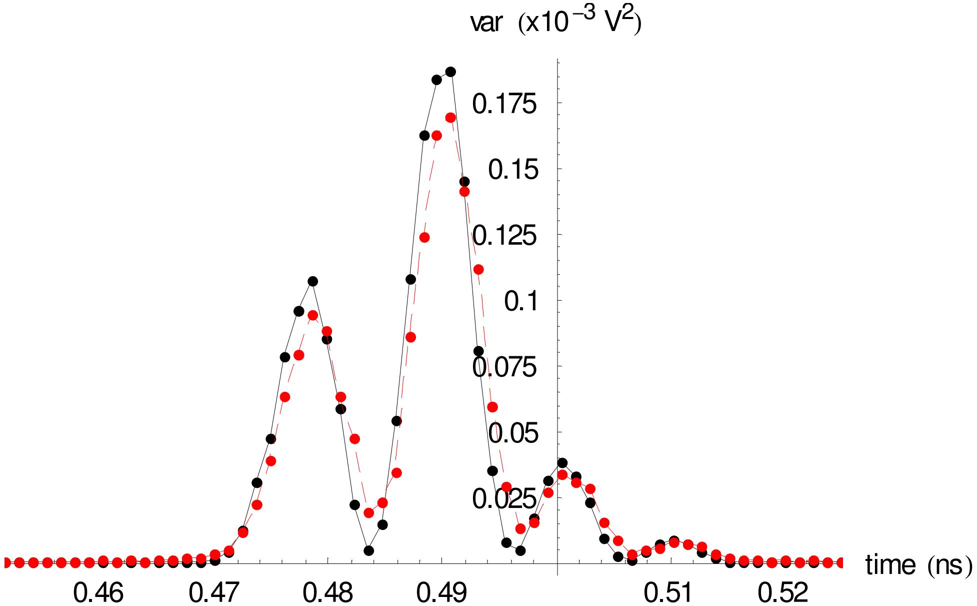
Shaping of variance by TBDt

1000 realizations of pulse → apply delay τ

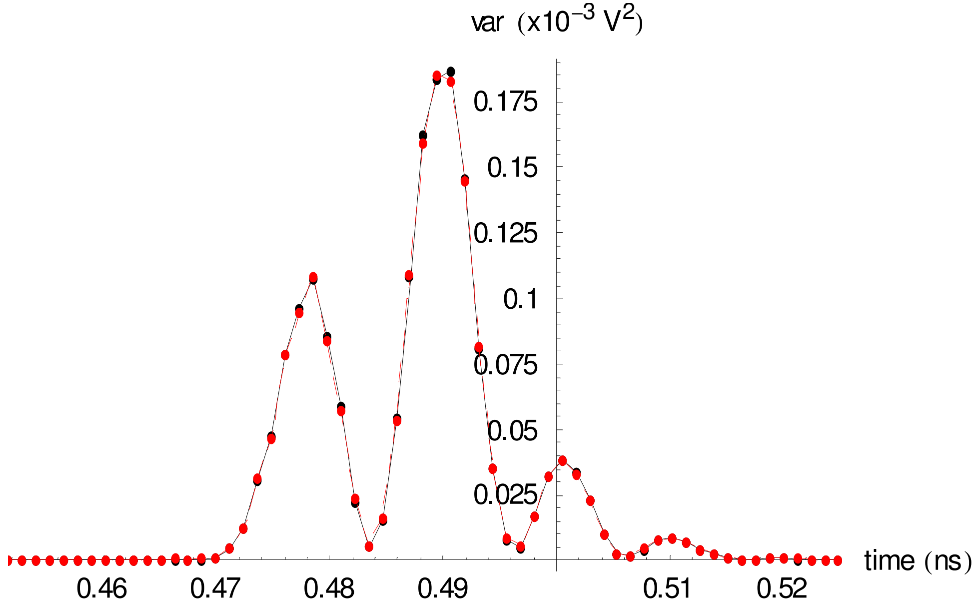
sample variance — **sample variance**

apply delay -τ

compare



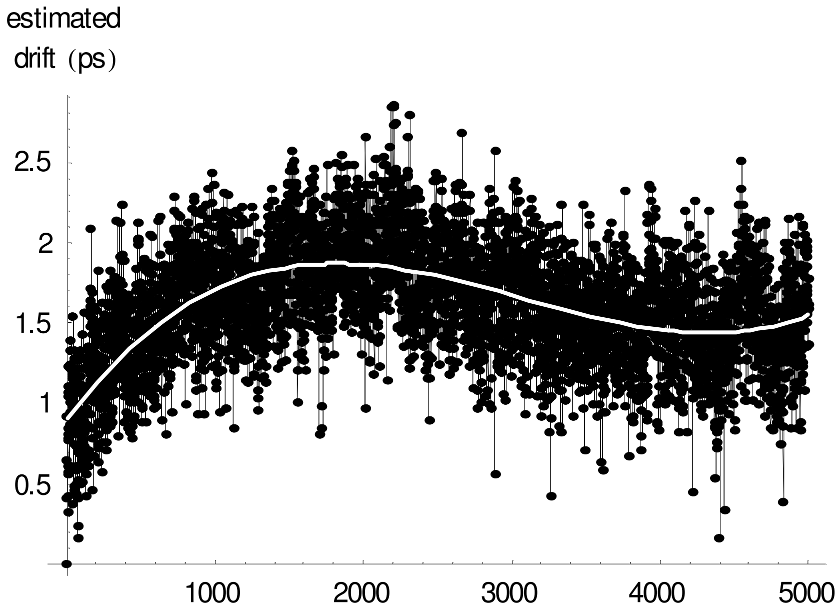
$\tau = \Delta t / 2$

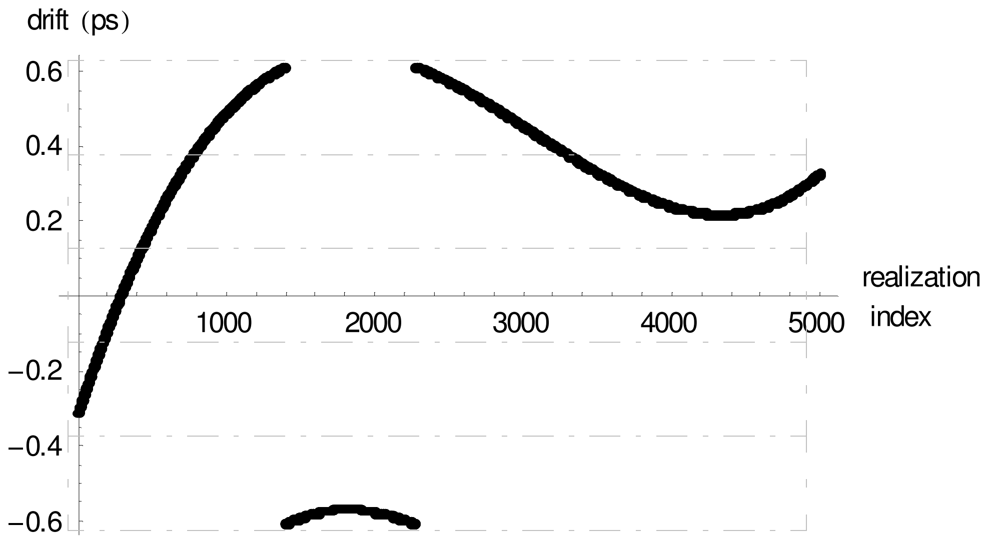


$\tau = \Delta t / 10$

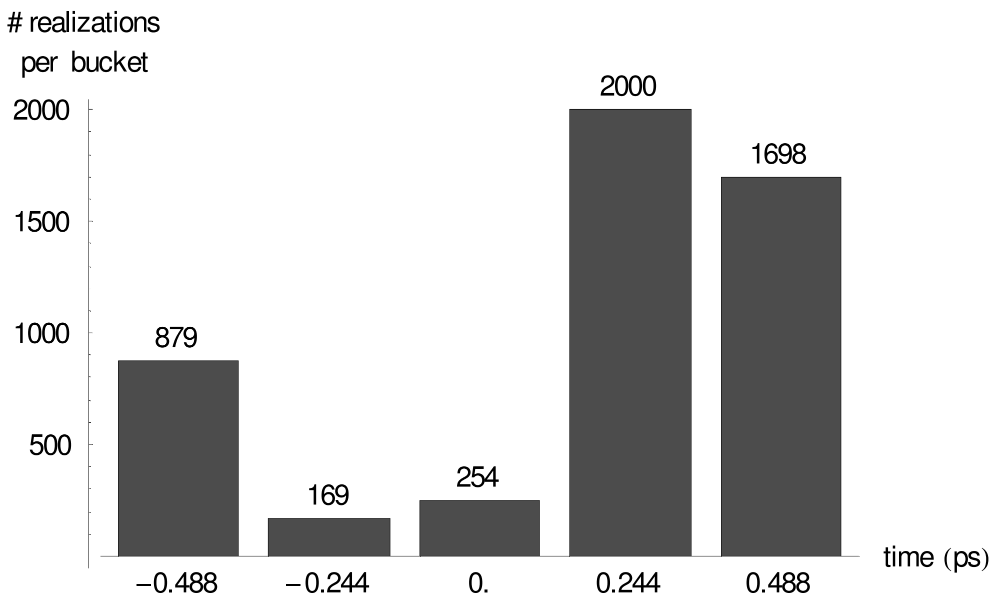
2-step solution

apply delay of k.Δt
align in buckets of Δt/5 wide



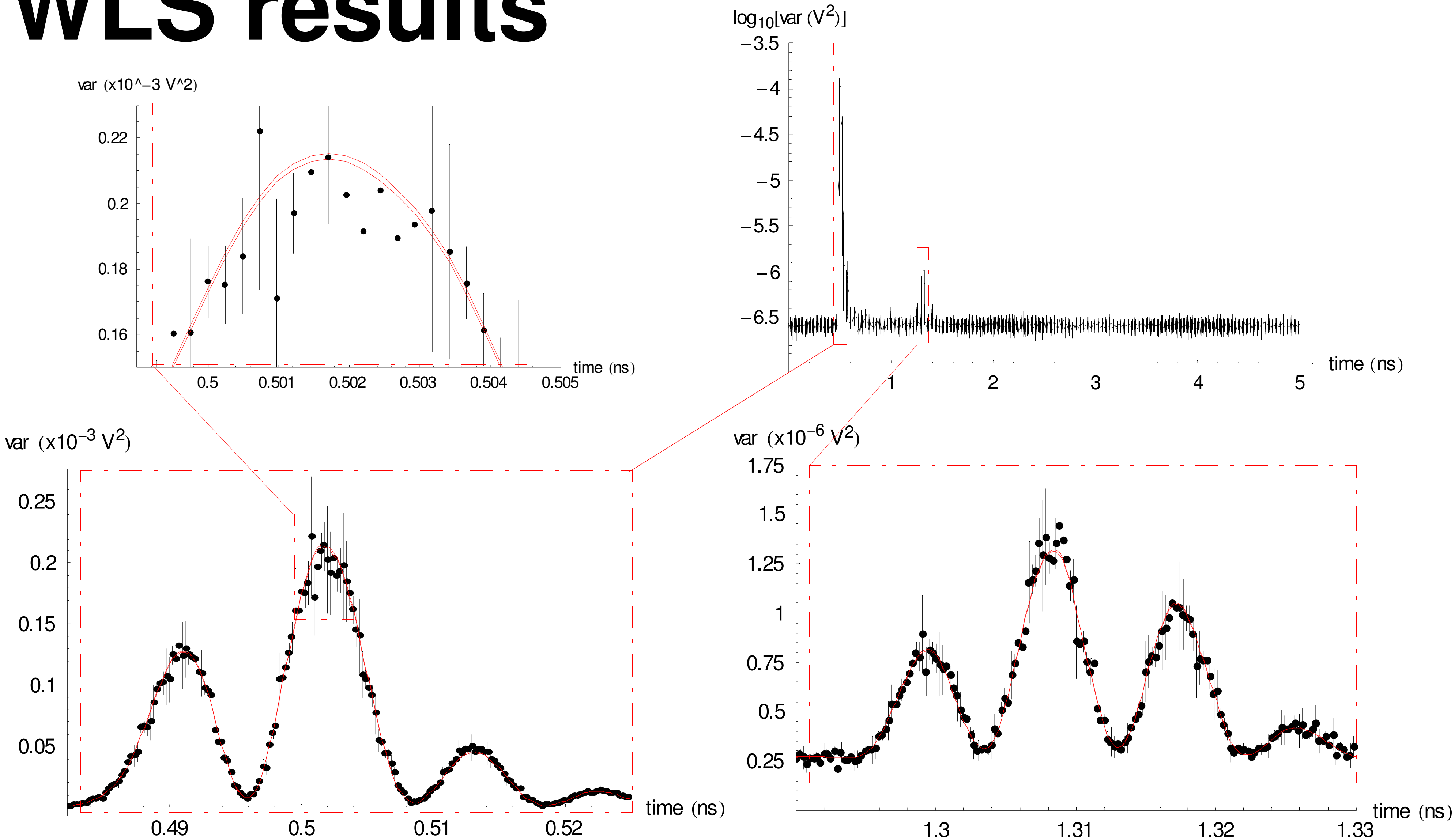


varying variance of sample variance



max. drift compensation = Δt/10

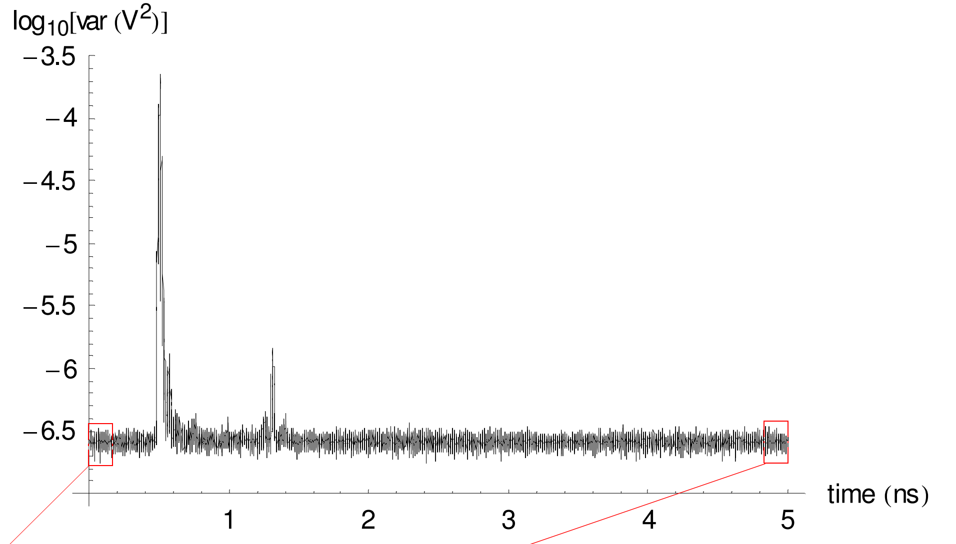
WLS results

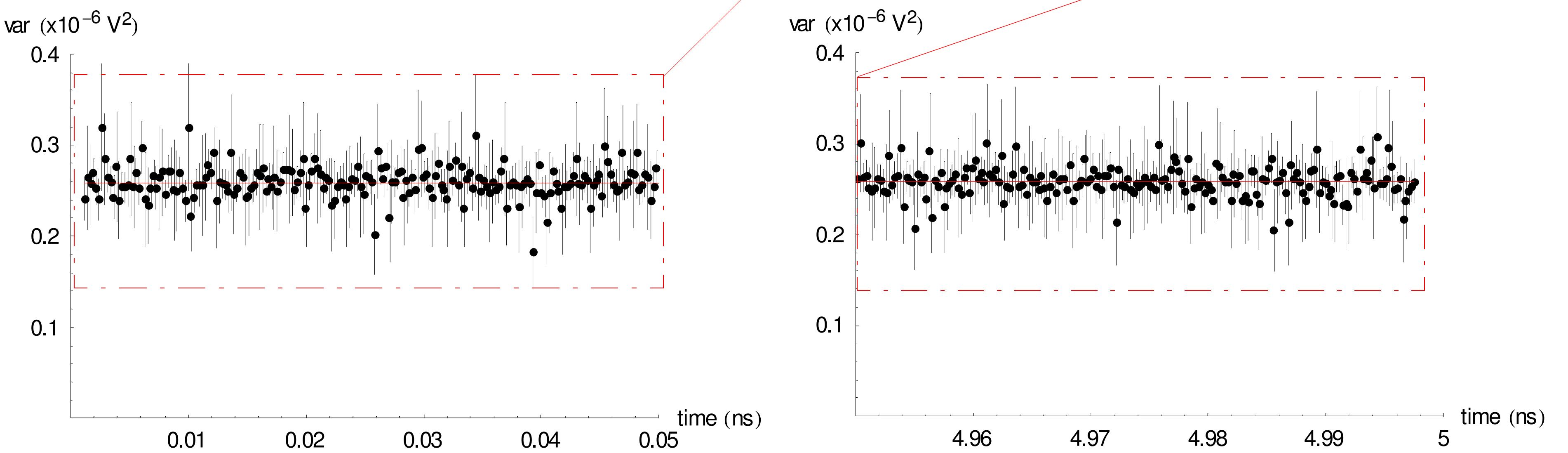


$$\sigma_{n_t} = 0.965 \pm 0.0023 \text{ ps}$$

WLS results

expected value cost:
20464 ± 397
realized value cost:
21830 = +7%





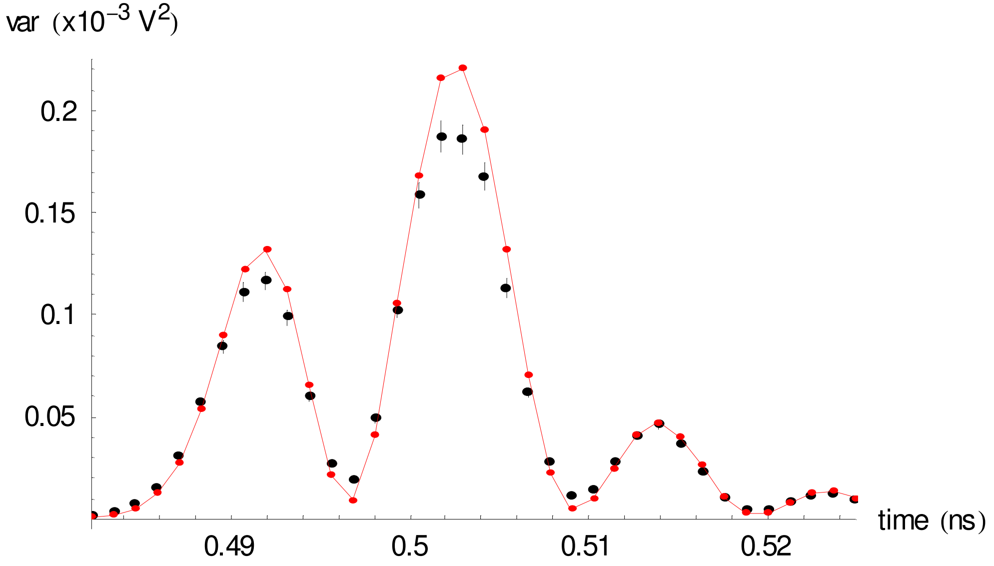
$$\sigma_{n_y} = \mathbf{0.508 \pm 0.00016\ mV}$$

The power of ...

What if we don’t limit drift compensation to $\Delta t/10$?

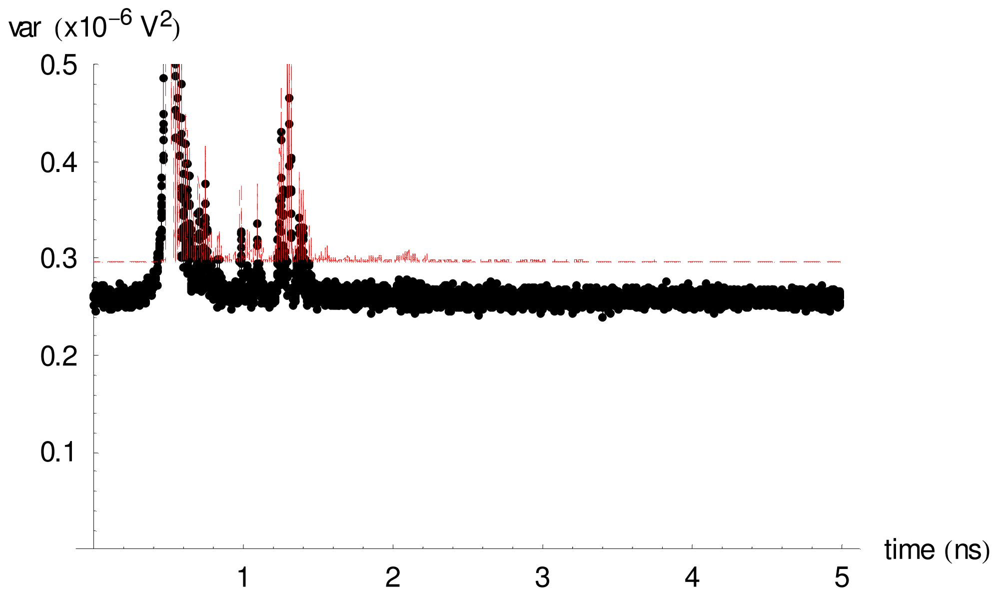
3rd order WLS

expected value cost: 4212 ± 188
realized value cost: 14764 (x 3.5)



1st order LS

bias of more than 10% on the estimate of the variance of the additive noise



Conclusions

- extension and/or enhancement of existing methods
- able to detect model errors and anomalies
- error bounds on estimates and modeled variance
- simultaneous estimation of
  variance of additive noise and jitter noise

- **system identification works !**
- **also at microwaves !**
- **and it's not that difficult !**