A Dynamic Model, Including Contact Bounce, of an Electrostatically Actuated Microswitch

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Abstract

Microelectromechanical devices are increasingly being integrated into electronic circuitry. One of these types of devices is the microswitch, which acts much like a three-terminal field effect transistor (FET). While various microswitches are currently being developed, their dynamic behavior is not well understood. Upon closing, switches bounce several times before making permanent contact with the drain. In this paper, a time-transient finite difference analysis is used to model the dynamic behavior of two different electrostatically actuated microswitch configurations. The model uses dynamic Euler-Bernoulli beam theory for cantilevered beams, includes the electrostatic force from the gate, takes into account the squeeze-film damping between the switch and substrate, and includes a simple spring model of the contact tips. The model and simulation can be used as design tools to improve switch performance and reduce switch bounce in future designs.

1. Introduction

Microelectromechanical devices are increasingly being integrated into electronic circuitry as their benefits become apparent. One of these types of devices is the microswitch, which acts much like a three-terminal FET. Two microswitch designs are considered here – a uniform width switch and a nonuniform width switch. Each is based on a cantilever beam, positioned above a substrate, with two small tips at the free end located directly above the drain (Figure 1). The gate is fabricated on the substrate and is generally placed under the mid and end portions of the beam. A voltage applied between the gate and the beam creates an electrostatic force which draws the beam downward causing the tips to contact the drain and allowing current to flow. We have found that the performance of the uniform width switch can be improved by changing the shape of its cross-section. A recent version of this nonuniform switch, that was measured for the work described here, is shown in Figure 2.

There are several attractive characteristics of microswitches. The devices described in [1] are surface micro-machined gold switches on a silicon substrate which are fabricated using the NUMEM process [2]. These switches exhibit on-resistances of less than 1 ohm and off-resistances greater than 10^{12} ohms, with lifetimes greater than 10^{9} cycles under nitrogen ambient conditions. Micromachined relays can be fabricated in large numbers on a single die which may contain other electronic devices. The lack of high temperature steps in the fabrication process means that the relays can be included as post-process additions to a conventional integrated circuit.

Micromachined devices can potentially preserve the high integration levels of solid-state switches but offer electrical performance similar to electromagnetic relays. Initial applications for the microswitch technology may include audio and video crosspoint switches, transmit/receive switches for wireless applications, multiplexers for data acquisition, switch arrays, and calibration trees. For example transmit/receive switching would be accomplished with lower losses and at far lower powers than are achievable with p-i-n switching. A video crosspoint switch would have much lower distortion and loss than its FET equivalent, and would be a purely passive device, without the requirement to re-amplify the switched signal.

Microswitches also have the potential to surpass the established operating temperature range for traditional solid state devices. Disadvantages include hysteresis for certain switch geometries and slower switching speed than traditional semiconductor devices.

Certain aspects of microswitch operation are not fully understood. Due to beam dynamics, switches do not remain closed the first time they contact the substrate. It is the dynamic behavior of microswitches that is the subject of this investigation. Specifically, when a switch closes, it bounces several times before making permanent contact with the drain. It is known that switch lifetime decreases with the number of switching cycles, although the precise mechanism responsible for this phenomenon is not well understood [1]. Thus switch bouncing, which subjects the beam and tips to multiple contact cycles for each complete switching cycle, may also decrease switch lifetime. Switch bouncing also increases the total amount of time between the instant at which the actuating voltage is applied and the instant when current is allowed to flow without interruption.

This paper presents a mathematical model of the dynamic behavior of a microswitch using the finite difference method. Previously, static analyses have been performed to determine the beam equilibrium position for a given voltage using both lumped parameter and finite difference methods [1-3], but these models cannot capture dynamic behavior, such as bouncing and switching time. Design and optimization of MEMS devices requires efficient solution techniques [4]. Reduced order macro-models created from finite element simulations [5] have been used to model the dynamic behavior of other MEMS devices. In the current investigation, an efficient dynamic finite difference analysis is implemented to provide an understanding of microswitch behavior, including switch bouncing, during the closing process [6].

2. Model Description

The development of the finite difference model can be divided into four components. The dynamic beam equations are combined with the electrostatic force equation and with a specially developed squeeze-film damping formulation. Finally, the tip-spring boundary conditions are included in the model to account for the impact of switch-tips with the substrate.

2.1 Governing Equation for Beam Deflection

The first microswitch configuration is modeled using Euler-Bernoulli beam theory with a constant cross-sectional area along the length of the beam (Figure 1). The equations of motion are

$$-EIy'''' + f_e - P^* = m\ddot{y}$$

where (1)

$$\ddot{y} = \frac{\partial^2 y}{\partial t^2}, \quad y'''' = \frac{\partial^4 y}{\partial x^4}$$

In (1), $y(x,t)$ is the downward deflection of the beam, f_e is the electrostatic force per unit length, P^* is the force per unit length provided by the squeeze-film between the switch and substrate, m

is the mass per unit length of the beam, E is Young's modulus, and I is the second moment of the cross-sectional area of the beam.

A solution of the governing differential equation will now be determined. Due to the non-linearities in the electrostatic force and in the squeeze-film damping (described in sections 2.2 and 2.3), an analytical solution is impractical to obtain and a finite difference numerical solution with respect to both x and t is sought. Due to the non-linearity, an explicit method with respect to time is utilized. This method should provide accurate results as long as the scales of the elements and time steps are sufficiently small.

The solution to (1) is found using standard central finite difference approximations for $\partial^4 y/\partial x^4$ and $\partial^2 y/\partial t^2$. The four spatial boundary conditions needed to find a solution are determined from the requirement that one end of the beam is clamped (zero displacement and zero slope) and the other end is free (zero bending moment and zero shear force). These conditions allow (1) to be expressed solely in terms of the nodal deflections (y_i, i = 1,2,…N), where node *1* is located at the fixed end of the beam and node N is located at the beam tips. The boundary conditions are

$$\begin{aligned} y(0) = 0 &\Rightarrow y_1 = 0 \\ y'(0) = 0 &\Rightarrow y_0 = y_2 \\ M(L) = 0 &\Rightarrow y_{N+1} = 2y_N - y_{N-1} \\ V(L) = 0 &\Rightarrow y_{N+2} = 4y_N - 4y_{N-1} + y_{N-2} \end{aligned} \tag{2}$$

The grid points represented by the displacements y_0, y_{N+1}, and y_{N+2} are image points needed for the finite difference approximations of the boundary conditions and field equations.

2.2 Electrostatic Force

The electrostatic force is the force per unit length that acts on the beam in the region directly above the gate. A node is placed at each end of the gate in the model. Thus

$$
\begin{aligned}
f_e &= \frac{\varepsilon_o V_o^{\,2} w}{4\left(d - y_i^{\,t}\right)^2} \quad , && i = i_{start}, i_{end} \\
f_e &= \frac{\varepsilon_o V_o^{\,2} w}{2\left(d - y_i^{\,t}\right)^2} \quad , && i = i_{start} + 1 < i < i_{end} - 1 \\
f_e &= 0 \,, && i = i < i_{start}; \quad i > i_{end}
\end{aligned}
\tag{3}
$$

The parameter ε_o is the permittivity of free space, V_o is the applied voltage between the beam and the gate, d is the initial beam-to-gate spacing, and w is the beam width. The subscripts "start" and "end" represent the beginning and end of the gate respectively. The superscript "t" indicates that y_i is evaluated at time t.

In the model the applied voltage is expressed as a fraction of the static threshold voltage. The static threshold voltage was calculated using the computer program of Majumder [3], which utilizes iteration in order to calculate the switch deflection for a given voltage using static beam bending theory. The voltage is increased by a specified increment in each loop step until the switch closes. The static thresholds were calculated for 11, 21, and 51 grid points using that program until convergence was achieved. The difference in the static threshold voltage between 21 and 51 grid points was 0.4% and 1.0% for the uniform and nonuniform switches respectively.

2.3 Squeeze-Film Damping

The squeeze-film damping pressure, due to the air film between the beam and the substrate, is determined using the simplified form of the Navier-Stokes equation known as the Reynolds equation. The Reynolds equation assumes that the viscous and pressure forces in the fluid film dominate the inertial terms. For the purposes of determining the squeeze-film pressure, the movement of the beam toward the substrate can be described as normally approaching surfaces. For this type of relative motion, the beam sliding velocity is equal to zero and the Reynolds equation can be reduced to [7]

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial z}\right)=\frac{\partial(\rho h)}{\partial t} \tag{4}$$

where $h = d - y$ is the beam-to-substrate spacing, and μ=1.86x10^{-5} N⁀s/m^2 (at 30^{0}C) and ρ are the viscosity and density of air respectively. This equation can be solved implicitly to determine the damping force at each node and time step, once the z-dependence has been removed. This simplification is accomplished by assuming that the pressure *p(x,z)* is a separable function in *x* and *z,* i.e.

$$p(x,z) = P(x)\left(1-\frac{4z^2}{w^2}\right) \tag{5}$$

where *w* is the beam width and *P(x)* is the pressure acting on the beam axis *(z=0).* Substituting (5) into (4), assuming that ρ and μ are constant throughout the fluid and that *h* is independent of *z,* and integrating across the width of the beam yields

$$2wh^2\frac{\partial h}{\partial x}\frac{\partial P}{\partial x}+\frac{2wh^3}{3}\frac{\partial^2 P}{\partial x^2}-\frac{8h^3}{w}P(x)=12w\mu\frac{\partial h}{\partial t} \tag{6}$$

Although this equation is non-linear in *h,* for specified *h* equation (6) is linear in *P*. Thus a solution is readily obtainable and is used to find the damping pressure at each time step.

The explicit formulation for the beam governing equation (1), requires only the beam configuration at time "t" and the squeeze-film pressures at time "t" to calculate the beam displacement at the next time "t+1". Thus, the beam displacement at "t+1" can be used to calculate the damping pressure at "t+1" using equation (6). This procedure provides the damping pressure for the next set of beam calculations. The only drawback is that the backward derivative approximation for $\partial y/\partial t$ must be used in the damping formulation. This representation is not quite as accurate as using the central difference approximation, but should be sufficient due to the scale of the time steps necessary for the explicit solution of (1). The standard central difference approximations for $\partial P/\partial x$, $\partial^2 P/\partial x^2$, and $\partial h/\partial x$ are also used in the formulation. The equations for the first and last nodes can be defined using the two boundary conditions needed to satisfy the second-order differential equation (6). The boundary conditions come from the requirement that the air cannot escape at the fixed end of the beam (because it is blocked by the support) and the condition that the air pressure at the beam tip must be atmospheric pressure. These boundary conditions are

$$\begin{aligned} &\left.\frac{\partial P}{\partial x}\right|_{x=0} = 0, \quad \Rightarrow P_o = P_2 \\ &P(L,t) = 0, \quad \Rightarrow P_N = 0 \end{aligned} \tag{7}$$

The solution to (6) provides the pressure at each node at time "t+1" but, before these expressions are substituted into (1), they must be converted to the force per unit length, P^*, that is used in (1). The conversion can be done by integrating p(x,z) across the width of the beam. The integration of (5) reveals that

$$P^*(x) = \frac{2w}{3} P(x) \tag{8}$$

With this modification, the solution to (6) can be substituted into (1) and a complete solution to the beam displacement equation determined. In order to start the solution process, the air pressure between the beam and gate is set equal to zero for the first time step.

2.4 Tip Spring Boundary Condition

A simple spring at the free end of the beam is used to model the effect of the two switch tips contacting the drain. Each tip acts essentially as a constraint on the free end of the beam. When the tips are in contact with the drain, the beam is no longer cantilevered, but is fixed at one end and spring-supported at the other end.

This modification is accomplished by inserting the spring force f_s as a boundary condition at the beam tip. The use of a spring force changes the last of the boundary conditions defined in the original set of conditions (2) such that the shear force at the end of the beam is now equal to the spring force. Thus $(2)_4$ becomes

$$V(L) = -f_s = -\frac{\partial M}{\partial x} = -EI\,y'''(L) \tag{9}$$

in which the finite difference approximation for $\partial^3 y/\partial x^3$ is used.

The spring force f_s is approximated by a linear spring model in which the spring compression is equal to the displacement of the beam tips less the initial spacing between the tips and the drain (d_T), i.e.

$$\begin{aligned} f_s &= k(y - d_T), && y > d_T \\ f_s &= 0, && y \le d_T \end{aligned} \tag{10}$$

The value of the spring constant k is *estimated* using a previously developed multi-asperity model that relates asperity deformation of the beam tips to the contact force [8]. The results of that model show that the tip stiffness is about 30 times greater than the beam stiffness for the uniform switch considered. It is noted that the use of a linear spring model to represent tip-to-drain contact is itself an approximation. The effects of contact non-linearities and adhesion have been neglected. Furthermore the surface topographies of the tip and drain, which are used to calculate the spring stiffness, are not well-known.

3. Model Results

3.1 Comparison With Free Vibration Analytical Solution

The first step in validating the model is to check the free vibration response of the beam. Typical switch properties and dimensions for the first switch configuration are used. In this case the beam length, width, and thickness are 70 μm, 30 μm, and 2 μm respectively. The tip height is 0.75 μm and the beam is initially 1.5 μm above the gate and the substrate, and is fabricated from gold-plated nickel (E = 207 GPa, ρ = 8900 kg/m^3). In this configuration the gate is positioned between 21 μm and 49 μm from the fixed end of the beam. The natural frequencies and mode shapes can be determined from the analytical solution [9] which yields the fundamental natural frequency of 317.98 kHz. The analytically determined first mode shape is then input into the numerical model as an initial condition. The model is run with a time step of 1 ns and the damping force, electrostatic force, and tip-spring force each set equal to zero. When 11 nodes are used, the resulting natural frequency is 315.32 kHz, which is 0.8% less than the analytical solution, and the mode shape is maintained. Similarly when 21 and then 51 nodes are used, the resulting natural frequencies are 317.34 kHz and 317.88 kHz which are only 0.2% and 0.03% respectively less than the analytical solution. When the electrostatic force and squeeze-film damping are included, the effect of element size may be somewhat more important, but the large increase in computational time does not warrant the use of a large number of nodes. Thus 21 nodes (corresponding to a grid mesh size of 5.35 μm) and a 1 ns time step were used in the simulations.

The accuracy of the model was also checked by artificially increasing the viscosity of air used in the calculations. With a sufficiently high viscosity, the beam should behave in a static manner. For a viscosity fifty times that of air under ambient conditions, the model results show that the switch closes when the applied voltage is equal to 99% of the static threshold voltage. As the viscosity was increased further, the closing voltage approached the static threshold voltage.

3.2 Model Results for an Uniform Switch

The full model is now examined for a uniform cross-section switch. The responses of the switch tip for three activation voltages ($1.2V_{th}$, $1.4V_{th}$, $1.6V_{th}$), where V_{th}=175.3V, are shown in Figure 3 where the drain position corresponds to a tip displacement of 0.75 μm. An increase in the applied voltage causes a decrease in the initial closing time and a decrease in the time at which the switch remains permanently closed. For $V=1.2V_{th}$ there are five bounces whereas for $V=1.6V_{th}$ there are three bounces. In general higher applied voltages result in a lower bouncing time (a smaller number of bounces and shorter duration bounces). Thus not only do the higher voltages cause the switch to close in a shorter time, but the reduction in bouncing allows the switch to carry an uninterrupted current at an earlier time. It is noted that, due to the compliance introduced by the tip-spring, the tip displacement is somewhat greater than the initial tip-to-drain spacing.

Figure 4 shows the tip displacements as a function of time for three smaller activation voltages ($0.8V_{th}$, $0.9V_{th}$, $1.0V_{th}$). For $V=0.8V_{th}$ the switch tip undergoes a damped oscillation without ever contacting the drain. At $V=0.9V_{th}$ the switch tip strikes the drain, bounces off and then undergoes a damping oscillation without again contacting the drain. For $V=V_{th}$ the tip contacts the drain and executes several bounces before making permanent contact after about 15 μs. It is also noted, although not shown in Figure 4, that the switch closes and remains closed after several bounces at about $0.979V_{th}$, indicating that the *dynamic* threshold voltage is 2.1% less than the *static* threshold voltage. This phenomenon is due to the combined effects of beam momentum (which causes a dynamic overshoot during actuation), and the nonlinearity introduced by the electrostatic force (3).

It is important to know the value of the minimum beam-to-gate spacing, because if that value is too small a gate short can occur. Thus Figure 5 shows the maximum *static* and *dynamic* displacements of the beam over the gate region. As expected the maximum displacement increases with applied voltage. However the effect of beam dynamics also increases substantially with applied voltage. At $V=V_{th}$ the maximum dynamic displacement is 15.6% greater than the corresponding static value, whereas for $V=1.7V_{th}$, the dynamic displacement is 56.2% greater than the static value. These effects are again due to a combination of beam momentum and the nonlinearity of the electrostatic force but are greater here than with the threshold voltage due to the dynamic overshoot which occurs after the initial contact of the tip with the drain. For voltages above $1.7V_{th}$ the beam moves rapidly toward the gate, indicating a dynamic snap-through behavior as the beam strikes the gate.

3.3 Model and Results for a Nonuniform Switch

Although the model was developed for a constant cross-section switch, it has been modified to analyze the nonuniform switch shown in the SEM micrograph in Figure 2 with the planar dimensions given in Figure 6. This switch is fabricated from gold (E = 80 GPa, ρ = 19320 kg/m^3). The side view of this switch is similar to that shown in Figure 1 with a thickness of 9.0 μm, d = 0.663 μm, and d_T = 0.362 μm. The gate has the same dimensions as the wide rectangular region of the switch (Figure 6) and is located directly beneath it. This geometry and material results in a lower threshold voltage and higher contact force [8]. The measured dynamic value of V_{th} was 37.5V whereas the corresponding calculated value was 35.0V. It is noted that the calculated static value of Vth was 40.3V, indicating that the effect of the momentum of the wide plate section was sufficient to cause the static and dynamic threshold values for this nonuniform switch to differ by 15%.

From the point of view of analysis, this configuration presents some challenges. First the beam now consists of two different cross-sectional areas. In order to adapt the model to analyze a two-section geometry using finite differences, four artificial nodes, two on each side of the interface between the two sections, are required. The two artificial nodes to the right of the interface represent an imaginary extension of the narrow beams while the two additional nodes to the left of the interface represent an imaginary extension of the wider beam. There are four additional continuity conditions which state that the displacement, slope, bending moment, and shear force are continuous across this interface.

More importantly, the small tip height (d-d_T=0.301 μm) results in a minimum spacing which is comparable to the mean-free-path of air (62 nm). When this configuration was simulated using the simplified Reynolds equation (6), the peak absolute pressure was over two atmospheres, indicating that the importance of compressibility. Thus the Reynolds equation (4) must be modified to include both compressibility and slip-flow terms due to molecular effects, as given by [7]

$$\frac{\partial}{\partial x}\left(h^3 p \frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3 p \frac{\partial p}{\partial z}\right)+6\lambda p_a \frac{\partial}{\partial x}\left(h^2 \frac{\partial p}{\partial x}\right)+6\lambda p_a \frac{\partial}{\partial z}\left(h^2 \frac{\partial p}{\partial z}\right)=12\mu \frac{\partial (ph)}{\partial t} \qquad (11)$$

where λ is the mean-free-path of air, p_a is atmospheric pressure, and p is now absolute pressure. Because the narrow beams are separated by a wide space, are relatively narrow, and are close to the fixed end of the beam, the effect of these beams on damping is neglected. The boundary conditions are modified so that the pressure is atmospheric along the perimeter of the plate region. Although the Reynolds equation (11) is two-dimensional, an explicit solution is possible by solving for $\frac{\partial p}{\partial t}$ in (11) and using a backward finite difference algorithm.

The modified model results for several voltages are shown in Figure 7. Again, the higher the actuation voltage, the faster the switch will initially close. However the smaller spacing and large rectangular area gives a higher damping force than the uniform switch previously analyzed. This increased damping increases the time to initial closing while decreasing the bounce amplitude and the number of bounces. The net effect is that this switch is usable after a shorter period of time has elapsed. Note however that the effect of increasing the actuation voltage is to increase the number and of bounces and the duration of bouncing. This trend is the reverse of what occurred for the uniform switch. The greater damping of the nonuniform switch causes the approach velocity of the contact tips to be lower than for the uniform beam. Thus dynamic bouncing is less prevalent for the nonuniform switch in cases in which the actuation voltage is close to the threshold voltage.

From the simulation results it is observed that the combined compression of the tip and the drain during impact can be as large as 90 nm (Figure 3) and 20 nm (Figure 7). This compression was smaller for the nonuniform switch due to the effect of the higher damping which decreases the approach velocity of the contact tips toward the drain. Also in both Figures 3 and Figure 7 there are high frequency oscillations after initial contact. Due to the limitations of the contact tip model discussed previously, these two phenomenon may not be modeled with a high degree of accuracy.

Because the gate has been placed near the beam tip, and due to the greater beam stiffness above the gate region, the minimum beam-to-gate spacing occurs at the beam tip. Thus this design is less susceptible to gate shorts than the uniform switch and allows a higher ratio of the applied voltage to the threshold voltage.

The large width of this design makes the use of a one-dimensional beam theory somewhat suspect. However until contact is made at the beam tips the loading is one-dimensional. After contact is made at the two contact tips, the effect of deflection variation in the *z*-direction (i.e. "bowing" deflections) should be accounted for in future work by using elastic plate theory.

3.4 Experimental Results

The model yields behavior which is similar to that seen in laboratory tests. Fabricated switches bounce several times before making permanent contact with the drain. Experimental results for switch bounce were obtained by measuring the voltage across the switch as a function of time with a Tektronix TDS 700A oscilloscope. The switch was placed in series with a 50Ω resistor and a 500mV source. The gate was driven with a pulse with a rise-time of approximately 50ns. The measurement circuit is shown in Figure 8. These measurements were done for the nonuniform switch geometry of the simulation just described and are shown in Figure 9. The trace with the single upward transition is the gate voltage. The other trace is the voltage measured across the switch. Because these switches were tested in room air, the resistance of the closed switch is approximately an order of magnitude higher than in switches tested in nitrogen.

A comparison between theory and experiment of the time to initial closure (T_1), the time to the beginning of the first bounce (T_2), and the time until the end of the first bounce i.e. the time to the beginning of the second closure (T_3), is shown in Figure 10. In determining these times from the simulation, bounces which were smaller than 5 nm in height were neglected. The experimentally determined times T_1, T_2 and T_3 were found from the traces of the switch voltage. These data points represent the transitions to/from voltages which are less than noise level of the switch, i.e. about 98% of the nominal switch voltage. The response time of the measurement circuit was sufficiently fast so as to have a negligible effect on the results shown in Figure 10. There is excellent agreement between the model and experiment for actuation voltages greater than 45V. The agreement for T_1 is better than for T_2 which is in turn better than for T_3. This behavior suggests that further improvements in modeling the impact dynamics of the contact tip upon the drain would lead to better predictions of the final closing time of the switch.

4. Conclusions

The dynamic behavior of an electrostatically actuated microswitch, including contact bounce, has been modeled using a time-transient finite difference method. The switch is modeled using Euler-Bernoulli beam theory, the damping force is calculated using the Reynolds Equation, and the tip-to-drain contact is modeled by a simple spring. Two configurations have been analyzed – a constant cross-section switch and a nonuniform switch. The minimum spacing in the latter case is small enough that molecular slip-flow terms and compressibility must be included in the Reynolds equation. After initially contacting the substrate, the switch undergoes a few bounces which last several microseconds until permanent contact is maintained. The closing time, the number and duration of the bounces, and the dynamic overshoot in the minimum beam-to-gate spacing, have been determined as functions of the switch geometry and material, and the applied voltage. Excellent agreement between theory and experiment with respect to the initial closing time and the duration of the first bounce has been obtained. The model and simulation can be used as design tools to improve switch performance and reduce bouncing in future designs. The effect of "bowing" deflections in the cross-directions and an improved model of the contact tip dynamics should be included in future work.

5. Acknowledgments

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7. List of Figures