Combinational Logic Review

Digital Devices was a **LONG, LONG** time ago in a galaxy **FAR, FAR, AWAY** for many of you.

We don't expect you to remember *everything* you learned in Digital Devices, but you need to remember > 0%.

We will review some to help you remember. You also need to go back and look at old notes. After a couple of days of review, we will expect you to be up to speed, and then we will ***ZOOM*** along.

Ask QUESTIONS during CLASS to SLOW things down.

Binary Representation

- The basis of all digital data is binary representation.
- Binary - means ‘two’
  - 1, 0
  - True, False
  - Hot, Cold
  - On, Off
- We must be able to handle more than just values for real world problems
  - 1, 0, 56
  - True, False, Maybe
  - Hot, Cold, LukeWarm, Cool
  - On, Off, Leaky

Number Systems

- To talk about binary data, we must first talk about number systems
- The decimal number system (base 10) you should be familiar with!
  - A digit in base 10 ranges from 0 to 9.
  - A digit in base 2 ranges from 0 to 1 (binary number system). A digit in base 2 is also called a ‘bit’.
  - A digit in base R can range from 0 to R-1
  - A digit in Base 16 can range from 0 to 16-1 (0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F). Use letters A-F to represent values 10 to 15. Base 16 is also called Hexadecimal or just ‘Hex’.

Positional Notation

Value of number is determined by multiplying each digit by a weight and then summing. The weight of each digit is a POWER of the BASE and is determined by position.

$953.78 = 9 * 10^2 + 5 * 10^1 + 3 * 10^0 + 7 * 10^{-1} + 8 * 10^{-2}$

$= 900 + 50 + 3 + .7 + .08 = 953.78$

$0b1011.11 = 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0 + 1*2^{-1} + 1*2^{-2}$

$= 8 + 0 + 2 + 1 + 0.5 + 0.25$

$= 11.75$

$0xA2F = 10*16^2 + 2*16^1 + 15*16^0$

$= 10 * 256 + 2 * 16 + 15 * 1$

$= 2560 + 32 + 15 = 2607$

Base 10, Base 2, Base 16

The textbook uses subscripts to represent different bases (ie. $A2F_{16}$, 953.78_{10}, 1011.11_{2})

I will use special symbols to represent the different bases. The default base will be decimal, no special symbol for base 10.

The ‘0x’ will be used for base 16 (0xA2F)
The ‘0b’ will be used for base 2 (0b10101111)

If ALL numbers on a page are the same base (ie, all in base 16 or base 2 or whatever) then no symbols will be used and a statement will be present that will state the base (ie, all numbers on this page are in base 16).

Common Powers

$2^{-3} = 0.125$
$2^{-2} = 0.25$
$2^{-1} = 0.5$
$2^{0} = 1$
$2^{1} = 2$
$2^{2} = 4$
$2^{3} = 8$
$2^{4} = 16$
$2^{5} = 32$
$2^{6} = 64$
$2^{7} = 128$
$2^{8} = 256$
$2^{9} = 512$
$2^{10} = 1024$
$2^{11} = 2048$
$2^{12} = 4096$

$16^{0} = 1 = 2^{0}$
$16^{1} = 16 = 2^{4}$
$16^{2} = 256 = 2^{8}$
$16^{3} = 4096 = 2^{12}$

$2^{10} = 1024 = 1$ Ki (kilobinary)
$2^{20} = 1048576 = 1$ Mi (1 megabinary) $= 1024$ K $= 2^{10} * 2^{10}$
$2^{30} = 1073741824 = 1$ Gi (1 gigabinary)

Conversion of Any Base to Decimal

Converting from ANY base to decimal is done by multiplying each digit by its weight and summing.

Binary to Decimal

$0b1011.11 = 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0 + 1*2^{-1} + 1*2^{-2}$

$= 8 + 0 + 2 + 1 + 0.5 + 0.25$

$= 11.75$

Hex to Decimal

$0xA2F = 10*16^2 + 2*16^1 + 15*16^0$

$= 10 * 256 + 2 * 16 + 15 * 1$

$= 2560 + 32 + 15 = 2607$

Conversion of Decimal Integer To ANY Base

Divide Number N by base R until quotient is 0. Remainder at EACH step is a digit in base R, from Least Significant digit to Most significant digit.

Convert 53 to binary

53/2 = 26, rem = 1 ← Least Significant Digit
26/2 = 13, rem = 0
13/2 = 6 , rem = 1
6 /2 = 3, rem = 0
3/2 = 1, rem = 1
1/2 = 0, rem = 1 ← Most Significant Digit

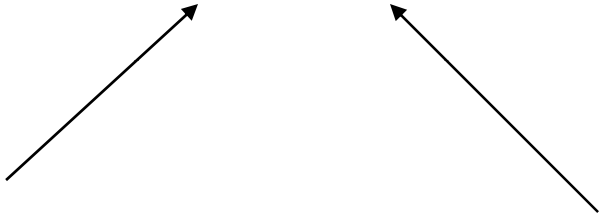
53 = 0b 110101

$= 1*2^5 + 1*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 1*2^0$

$= 32 + 16 + 0 + 4 + 0 + 1 = 53$

Least Significant Digit
Most Significant Digit

53 = 0b 110101



Most Significant Digit (has weight of 2^5 or 32). For base 2, also called Most Significant Bit (MSB). Always LEFTMOST digit.

Least Significant Digit (has weight of 2^0 or 1). For base 2, also called Least Significant Bit (LSB). Always RIGHTMOST digit.

More Conversions

Convert 53 to Hex

$53/16 = 3, \quad rem = 5$

$3/16 = 0, \quad rem = 3$

$53 = 0x35$

$= 3 * 16^1 + 5 * 16^0$

$= 48 + 5 = 53$

Hex (base 16) to Binary Conversion

Each Hex digit represents 4 bits. To convert a Hex number to Binary, simply convert each Hex digit to its four bit value.

Hex Digits to binary:

```
0x0 = 0b 0000
0x1 = 0b 0001
0x2 = 0b 0010
0x3 = 0b 0011
0x4 = 0b 0100
0x5 = 0b 0101
0x6 = 0b 0110
0x7 = 0b 0111
0x8 = 0b 1000
```

Hex Digits to binary (cont):

```
0x9 =  0b 1001
0xA =  0b 1010
0xB =  0b 1011
0xC =  0b 1100
0xD =  0b 1101
0xE =  0b 1110
0xF =  0b 1111
```

Hex to Binary, Binary to Hex

0xA2F = 0b 1010 0010 1111

0x345 = 0b 0011 0100 0101

Binary to Hex is just the opposite, create groups of 4 bits starting with least significant bits. If last group does not have 4 bits, then pad with zeros for unsigned numbers.

0b 1010001 = 0b 0101 0001 = 0x51

Padded with a zero

A Trick!

If faced with a large binary number that has to be converted to decimal, I first convert the binary number to HEX, then convert the HEX to decimal. Less work!

0b 110111110011 = 0b 1101 1111 0011

= D F 3

= $13 * 16^2 + 15 * 16^1 + 3*16^0$

= $13 * 256 + 15 * 16 + 3 * 1$

= $3328 + 240 + 3$

= 3571

Of course, you can also use the binary, hex conversion feature on your calculator. Too bad calculators won't be allowed on the first test, though…...

Binary Numbers Again

Recall than N binary digits (N bits) can represent unsigned integers from 0 to 2^N-1.

4 bits = 0 to 15
8 bits = 0 to 255
16 bits = 0 to 65535

Besides simply representation, we would like to also do arithmetic operations on numbers in binary form.
Principle operations are addition and subtraction.

Binary Arithmetic, Subtraction

The rules for binary arithmetic are:

0 + 0 = 0, carry = 0
1 + 0 = 1, carry = 0
0 + 1 = 1, carry = 0
1 + 1 = 0, carry = 1

The rules for binary subtraction are:

0 - 0 = 0, borrow = 0
1 - 0 = 1, borrow = 0
0 - 1 = 1, borrow = 1
1 - 1 = 0, borrow = 0

Borrows, Carries from digits to left of current of digit.

Binary subtraction, addition works just the same as decimal addition, subtraction.

Binary, Decimal addition

Decimal

34

\+ 17

51

from LSD to MSD:

7+4 = 1; with carry out of 1 to next column

1 (carry) + 3 + 1 = 5.

answer = 51.

Binary

0b 101011

\+ 0b 000001

101100

From LSB to MSB:

1+1 = 0, carry of 1

1 (carry)+1+0 = 0, carry of 1

1 (carry)+0 + 0 = 1, no carry

1 +0 = 1

0 + 0 = 0

1 + 0 = 1

answer = % 101100

Hex Addition

```
   0x3A
+  0x28
--------
    0x62
```

A+8 = 2; with carry out of 1 to next column

1 (carry) + 3 + 2 = 6.
answer = 0x62

Decimal check.

0x3A = 3 * 16 + 10
= 58
0x28 = 2 * 16 + 8
= 40
58 + 40 = 98

0x62 = 6 * 16 + 2
= 96 + 2 = 98!!

Hex addition again

Why is 0xA + 0x8 = 2 with a carry out of 1?

The carry out has a weight equal to the BASE (in this case 16). The digit that gets left is the excess (BASE - sum).

Ah + 8h = 10 + 8 = 18.

18 is GREATER than 16 (BASE), so need a carry out!

Excess is 18 - BASE = 18 - 16 = 2, so '2' is digit.

Exactly the same thing happens in Decimal.
5 + 7 = 2, carry of 1.
5 + 7 = 12, this is greater than 10!.
So excess is 12 - 10 = 2, carry of 1.

Subtraction

Decimal

```
  900
- 001
-------
  899
```

0-1 = 9; with borrow of 1 from next column
0 -1 (borrow) - 0 = 9, with borrow of 1
9 - 1 (borrow) - 0 = 8.
Answer = 899.

Binary

```
  0b 100
- 0b 001
  -------
      011
```

0-1 = 1; with borrow of 1 from next column
0 -1 (borrow) - 0 = 1, with borrow of 1
1 - 1 (borrow) - 0 = 0.
Answer = % 011.

Hex Subtraction

```
   0x34
-  0x27
----------
   0x0D
```

4-7 = D; with borrow of 1 from next column

3 - 1 (borrow) - 2 = 0.
answer = 0x0D.

Decimal check.

0x34 = 3 * 16 + 4
= 52
0x27 = 2 * 16 + 7
= 39
52 - 39 = 13

0x0D = 13 !!

Hex subtraction again

Why is 0x4 – 0x7 = 0xD with a borrow of 1?

The borrow has a weight equal to the BASE (in this case 16).

BORROW +0x4 – 0x7 = 16 + 4 - 7 = 20 - 7 = 13 = 0xD.

0xD is the result of the subtraction with the borrow.

Exactly the same thing happens in decimal.
3 - 8 = 5 with borrow of 1
borrow + 3 - 8 = 10 + 3 - 8 = 13 - 8 = 5.

Fixed Precision

With paper and pencil, I can write a number with as many digits as I want:

1,027,80,032,034,532,002,391,030,300,209,399,302,992,092,920

A microprocessor or computing system usually uses FIXED PRECISION for integers; they limit the numbers to a fixed number of bits:

0x	AF4500239DEFA231	64 bit number, 16 hex digits
0x	9DEFA231	32 bit number, 8 hex digits
0x	A231	16 bit number, 4 hex digits
0x	31	8 bit number, 2 hex digits

High end microprocessors use 64 or 32 bit precision; low end microprocessors use 16 or 8 bit precision.

Unsigned Overflow

In this class I will use 8 bit precision most of the time, 16 bit occasionally.

Overflow occurs when I add or subtract two numbers, and the correct result is a number that is outside of the range of allowable numbers for that precision. I can have both unsigned and signed overflow (more on signed numbers later)

8 bits -- unsigned integers 0 to 2^8 -1 or 0 to 255.

16 bits -- unsigned integers 0 to 2^{16}-1 or 0 to 65535

N bit – unsigned numbers 0 to 2^N-1

Unsigned Overflow Example

Assume 8 bit precision; ie. I can't store any more than 8 bits for each number.

Lets add 255 + 1 = 256. The number 256 is OUTSIDE the range of 0 to 255! What happens during the addition?

255 = 0x FF

\+ 1 = 0x 01

---

256 /= 0x00

/= means Not Equal

0xF + 1 = 0, carry out

0xF + 1 (carry) + 0 = 0, carry out

Carry out of MSB falls off end, No place to put it!!!

Final answer is WRONG because could not store carry out.

Unsigned Overflow

A carry out of the Most Significant Digit (MSD) or Most Significant Bit (MSB) is an OVERFLOW indicator for addition of UNSIGNED numbers.

The correct result has overflowed the number range for that precision, and thus the result is incorrect.

If we could STORE the carry out of the MSD, then the answer would be correct. But we are assuming it is discarded because of fixed precision, so the bits we have left are the incorrect answer.

Binary Codes (cont.)

N bits (or N binary Digits) can represent 2^N different values.

(for example, 4 bits can represent 2^4 or 16 different values)

N bits can take on unsigned decimal values from 0 to 2^N-1.

Codes usually given in tabular form.

000	black
001	red
010	pink
011	yellow
100	brown
101	blue
110	green
111	white

Codes for Characters

Also need to represent Characters as digital data.
The ASCII code (American Standard Code for Information Interchange) is a 7-bit code for Character data. Typically 8 bits are actually used with the 8th bit being zero or used for error detection (parity checking).
8 bits = 1 Byte.

‘A’ = % 01000001 = 0x41
‘&’ = % 00100110 = 0x26

7 bits can only represent 2^7 different values (128). This enough to represent the Latin alphabet (A-Z, a-z, 0-9, punctuation marks, some symbols like $), but what about other symbols or other languages?

ASCII

American Standard Code for Information Interchange

Most Significant Digit

	0x0	**0x1**	**0x2**	**0x3**	**0x4**	**0x5**	**0x6**	**0x7**
0x0	NUL	DLE	SPC	0	@	P	`	p
0x1	SOH	DC1	!	1	A	Q	a	q
0x2	STX	DC2	"	2	B	R	b	r
0x3	ETX	DC3	#	3	C	S	c	s
0x4	EOT	DC4	$	4	D	T	d	t
0x5	ENQ	NAK	%	5	E	U	e	u
0x6	ACK	SYN	&	6	F	V	f	v
0x7	BEL	ETB	'	7	G	W	g	w
0x8	BS	CAN	(	8	H	X	h	x
0x9	TAB	EM	)	9	I	Y	i	y
0xA	LF	SUB	*	:	J	Z	j	z
0xB	VT	ESC	+	;	K	[	k	{
0xC	FF	FS	,	<	L	\	l	|
0xD	CR	GS	-	=	M	]	m	}
0xE	SO	RS	.	>	N	^	n	~
0xF	SI	US	/	?	O	_	o	DEL

UNICODE

UNICODE is a 16-bit code for representing alphanumeric data.
With 16 bits, can represent 2^{16} or 65536 different symbols.
16 bits = 2 Bytes per character (the extended version uses 32-bits per character, or 4 bytes, for 4,294,967,296 different symbols).

0x0041-005A A-Z
0x0061-4007A a-z

Some other alphabet/symbol ranges

0x3400-3d2d	Korean Hangul Symbols
0x3040-318F	Hiranga, Katakana, Bopomofo, Hangul
0x4E00-9FFF	Han (Chinese, Japanese, Korean)

UNICODE used by Web browsers, Java, most software these days.

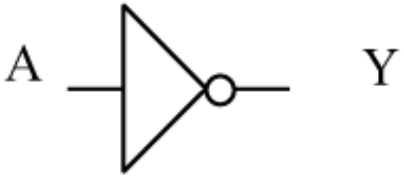
Number System Practice

What should you practice?

- Hex to decimal, decimal to hex conversion
- Hex to binary, binary to hex conversion
- Hex addition, subtraction

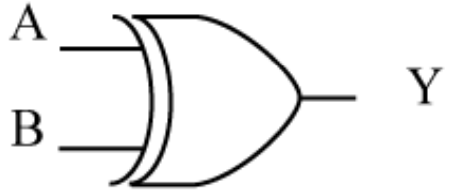
Basic Logic Gates

A	Y
0	1
1	0



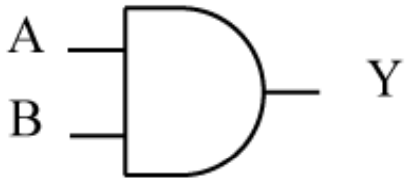
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

XOR



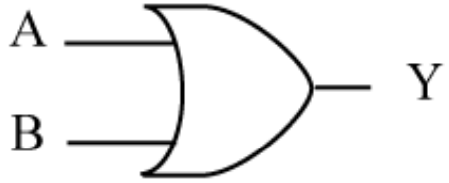
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

AND



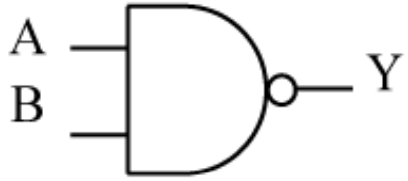
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

OR



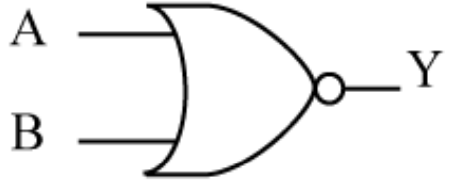
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NAND



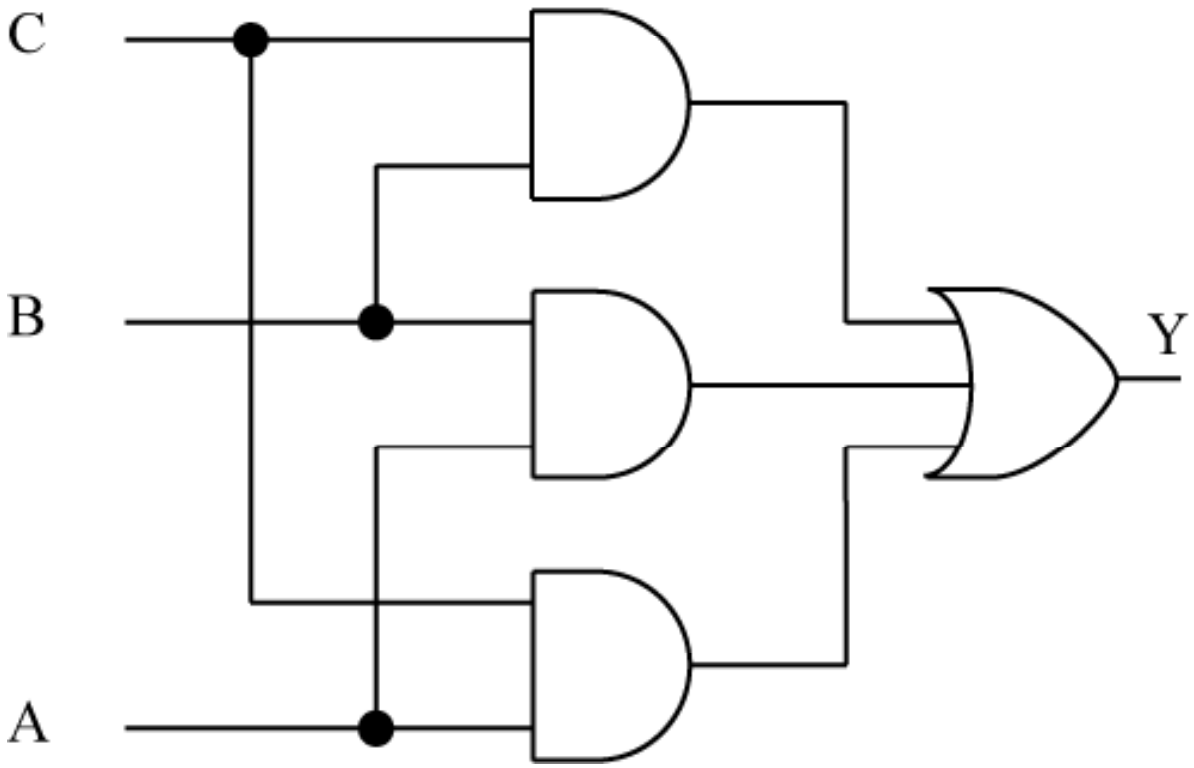
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

NOR



Majority Gate (and-or) form

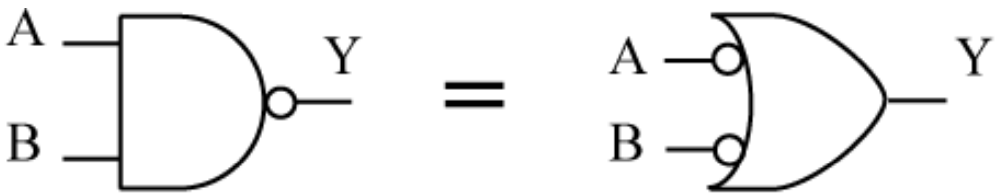
Majority Y = (A & B) | (B & C) | (A & C)



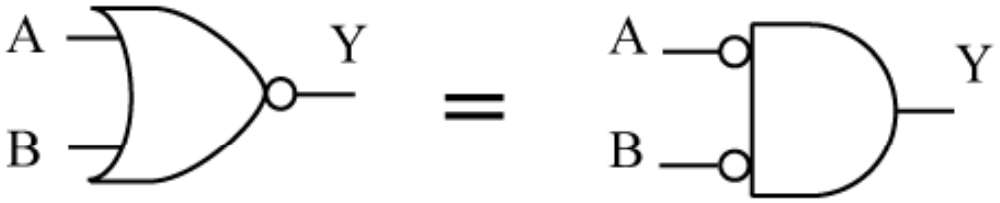
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

DeMorgan's Law

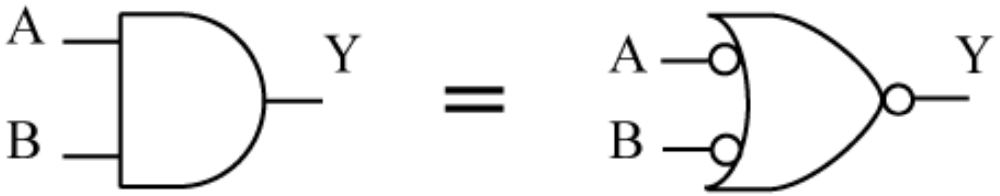
(1) $\sim(A \,\&\, B) = (\sim A) \mid (\sim B)$



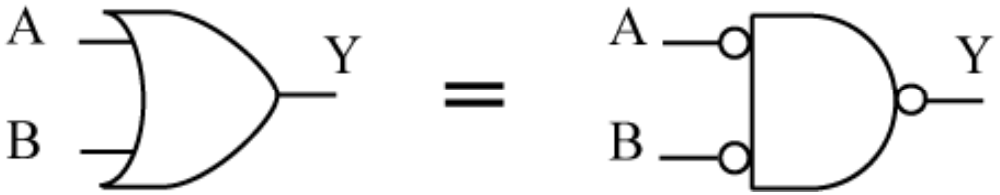
(2) $\sim(A \mid B) = (\sim A) \,\&\, (\sim B)$



(3) $A \,\&\, B = \sim((\sim A) \mid (\sim B))$



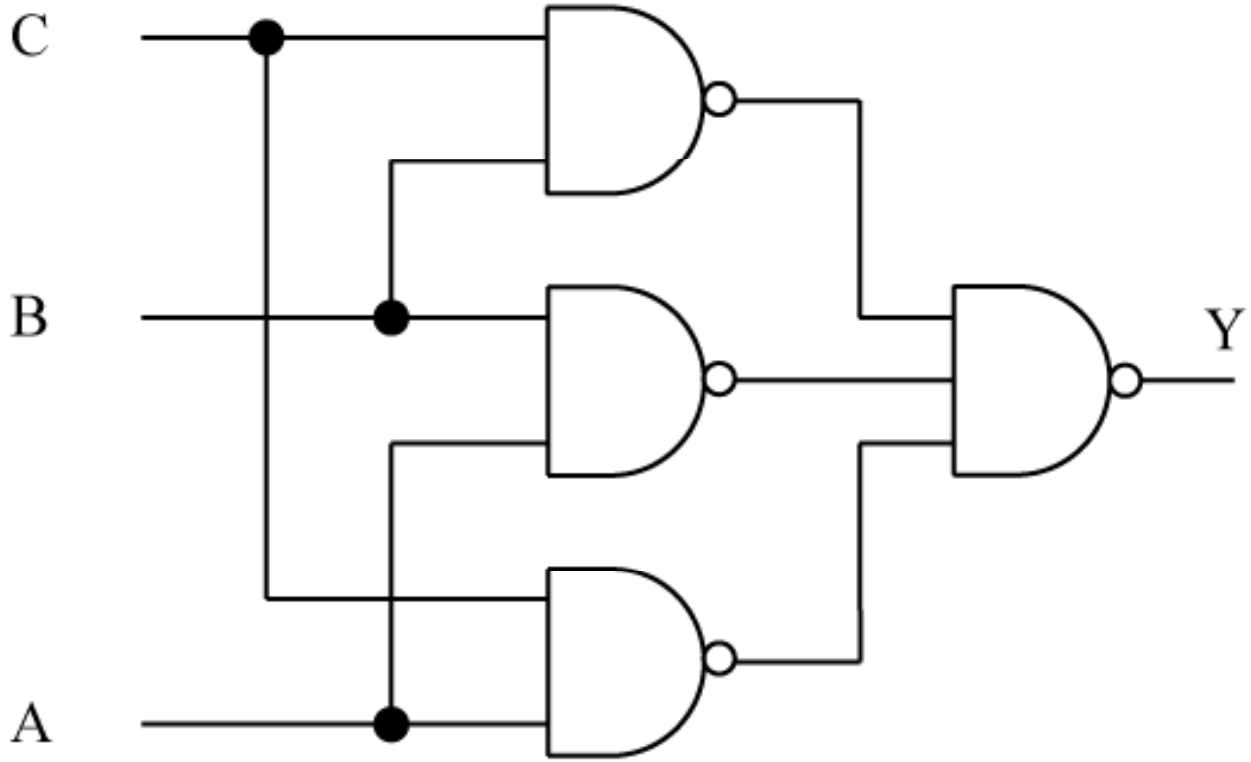
(4) $A \mid B = \sim((\sim A) \,\&\, (\sim B))$



Majority Gate (nand-nand) form

Majority

$$Y = \sim(\sim(A \,\&\, B) \;\&\; \sim(B \,\&\, C) \;\&\; \sim(A \,\&\, C))$$
$$= (\sim(\sim(A \,\&\, B)) \mid (\sim(\sim(B \,\&\, C))) \mid (\sim(\sim(A \,\&\, C)))$$
$$= (A \,\&\, B) \mid (B \,\&\, C) \mid (A \,\&\, C)$$

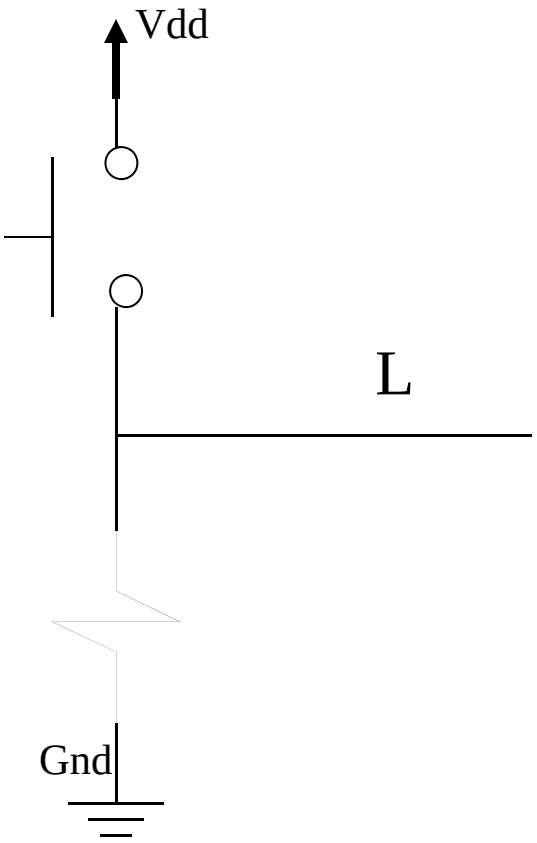


Representing ‘1’ and ‘0’

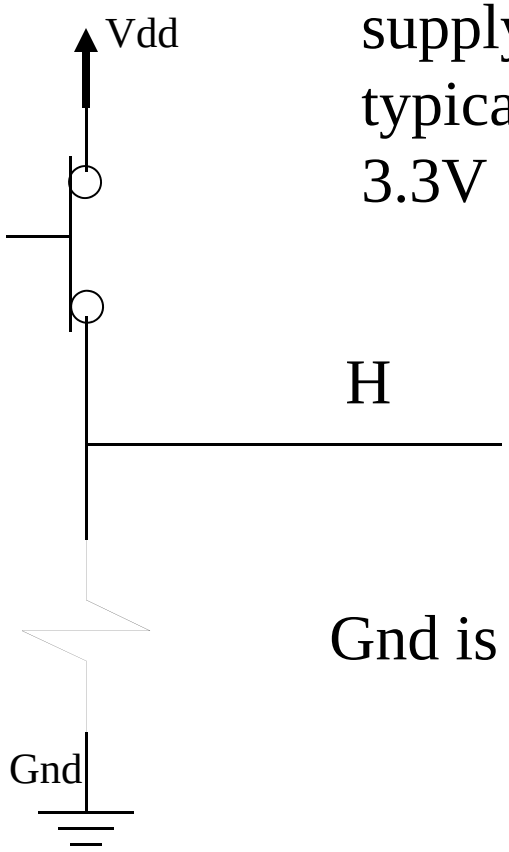
- In the electrical world, two ways of representing ‘0’ and ‘1’ are (these are not the only ways):
  - Presence or absence of electrical current
  - Different Voltage levels
- Different voltage levels are the most common
  - Usually 0v for logic ‘0’, some non-zero voltage for logic ‘1’ (I.e. > 3 volts)
- Can interface external sources to digital systems in many ways
  - Switches, buttons, other human controlled input devices
  - Transducers (change a physical quantity like temperature into a digital quantity).

Switch Inputs

High True switch



Switch open (**negated**), output is L

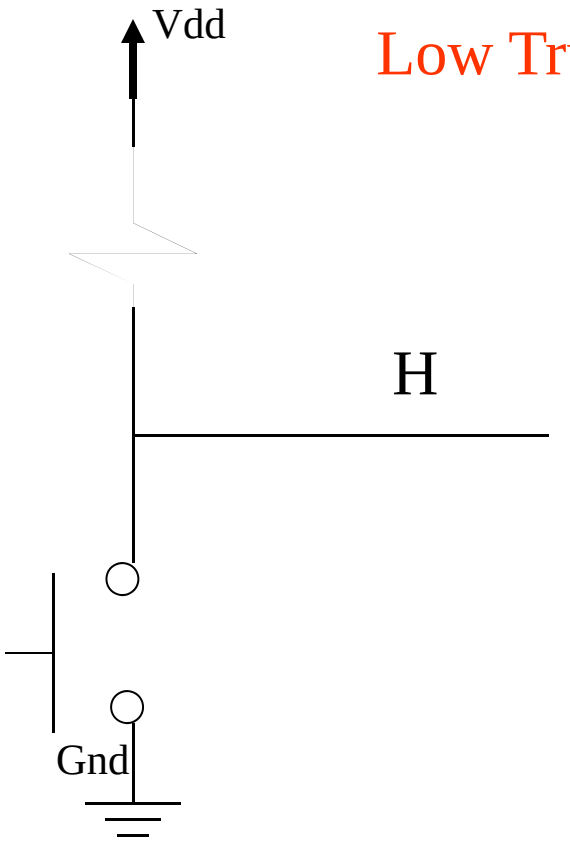


Switch closed (**asserted**), output is H

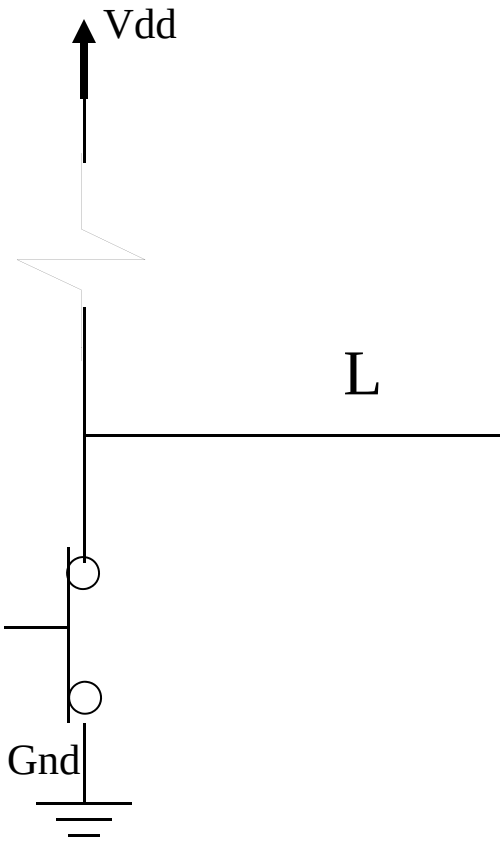
Vdd is power supply voltage, typically 5V or 3.3V

Gnd is 0 V

Examples of high, low signals

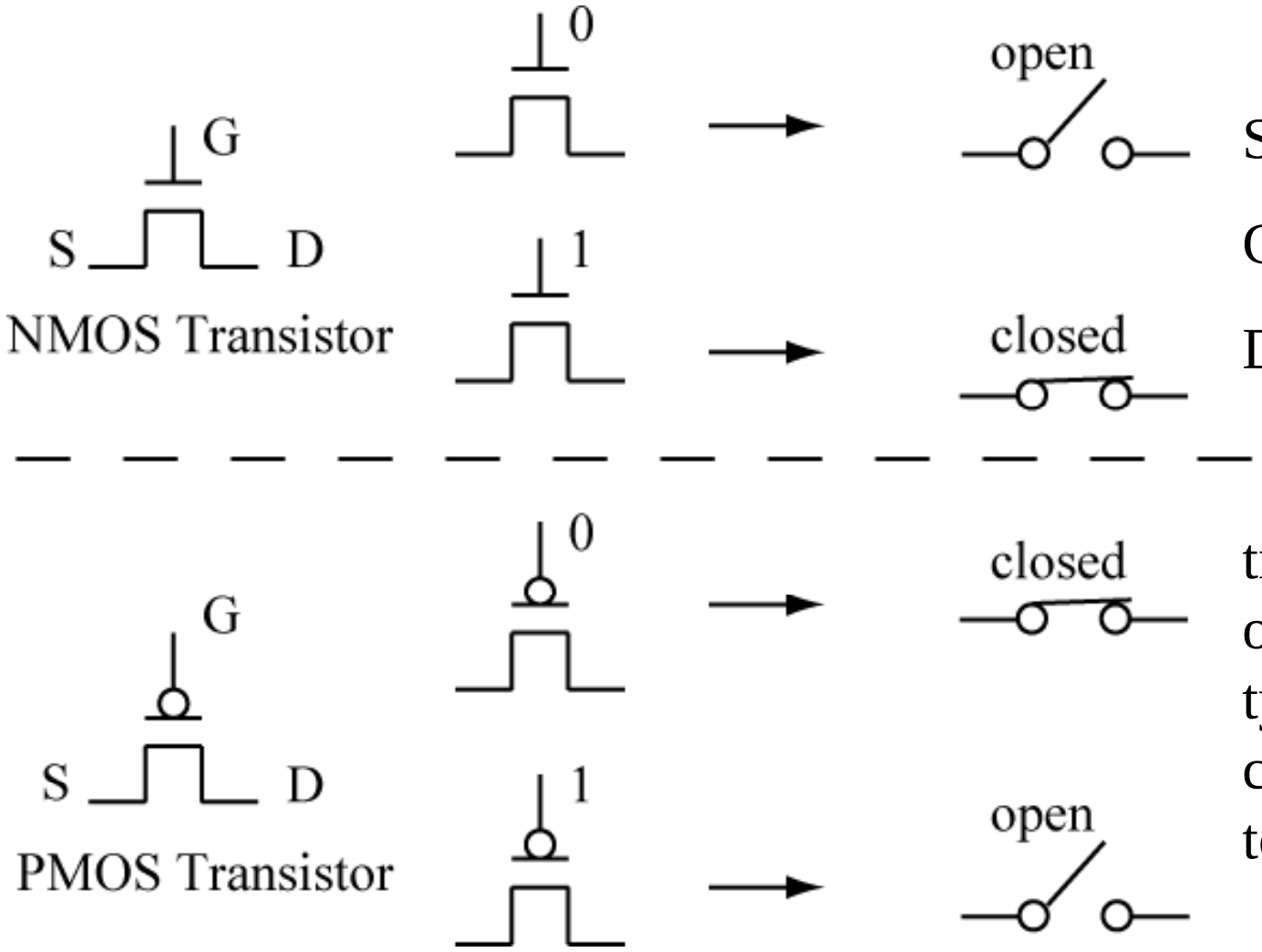


Switch open (negated), output is H

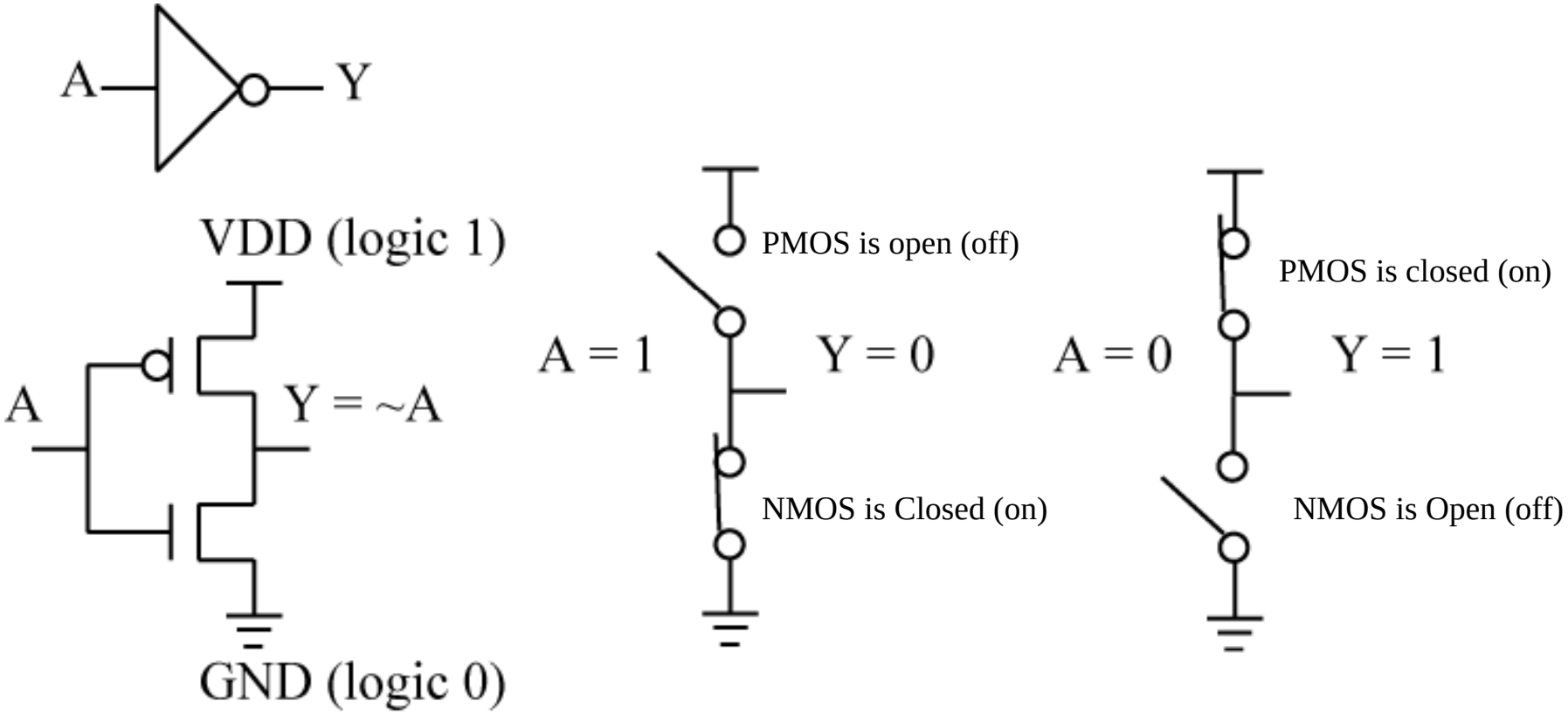


Switch closed (asserted), output is L

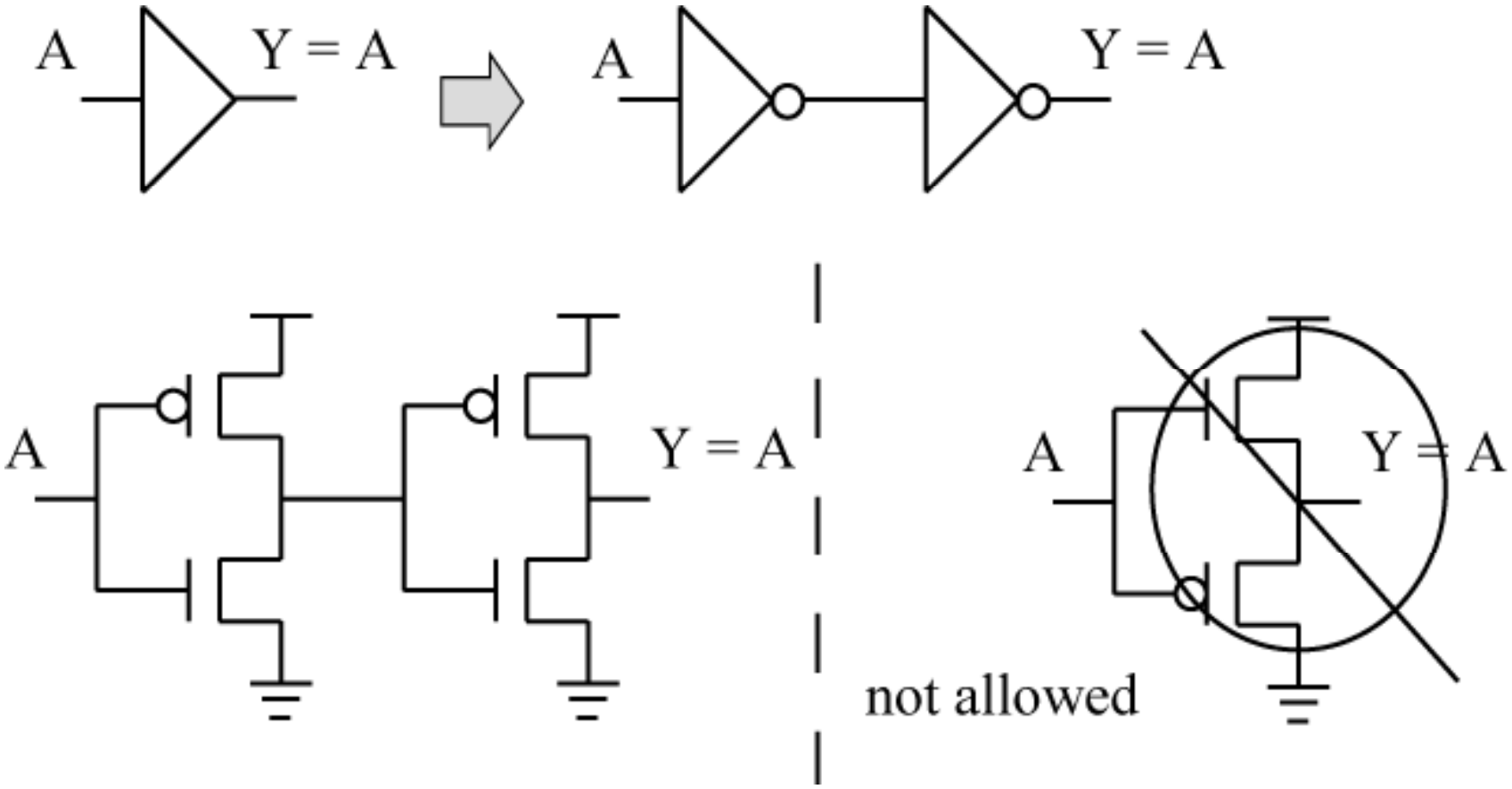
CMOS transistors (P, N)



Inverter gate - takes 2 transistors



Buffer - takes 4 transistors

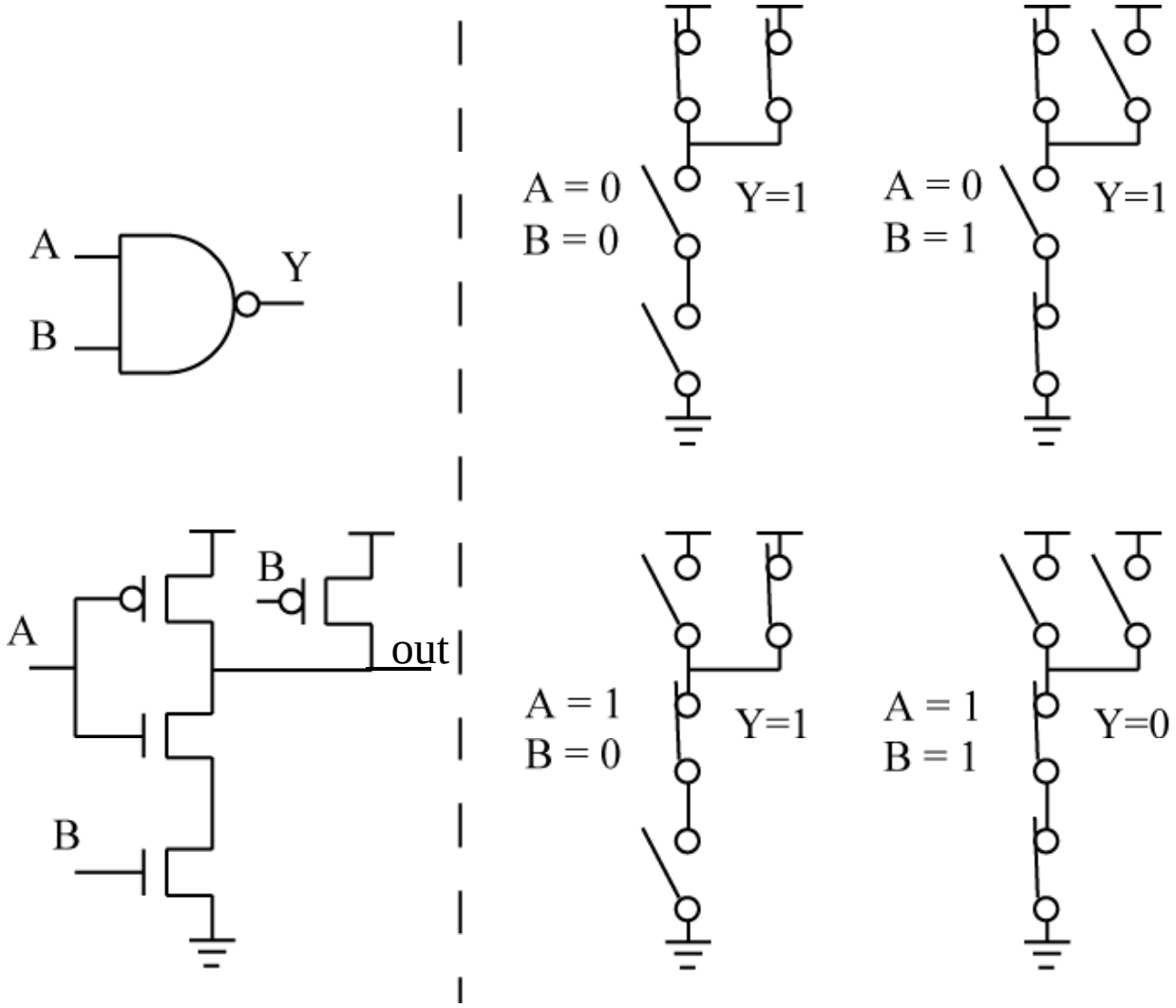


In digital logic, NMOS must be connected to ground, PMOS to VDD.

NAND gate - takes 4 transistors

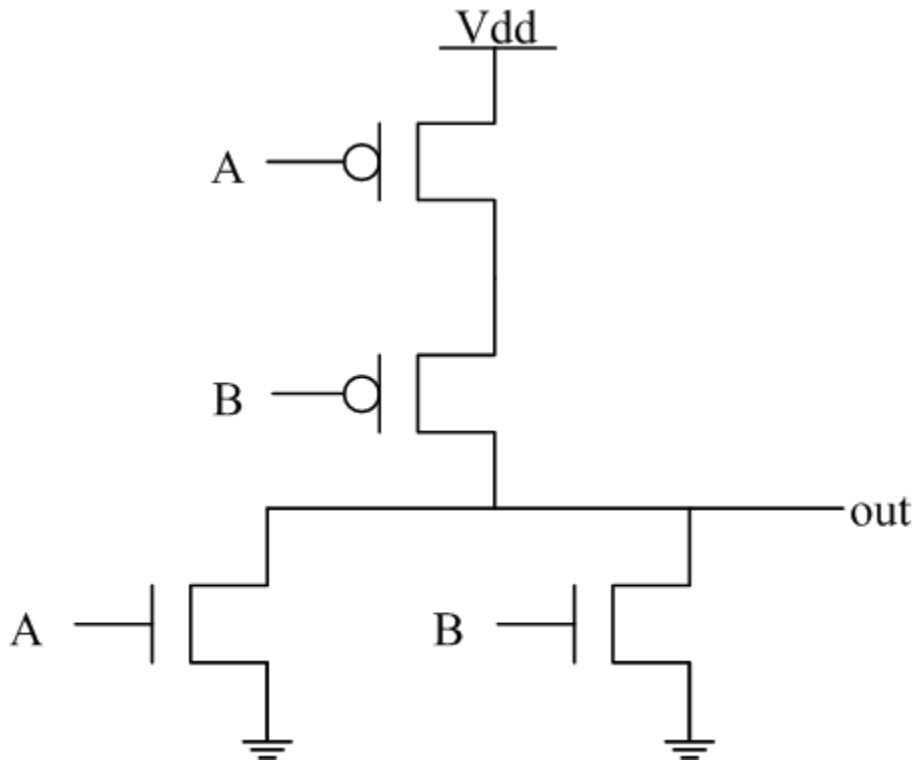
A	B	Y
L	L	H
L	H	H
H	L	H
H	H	L

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



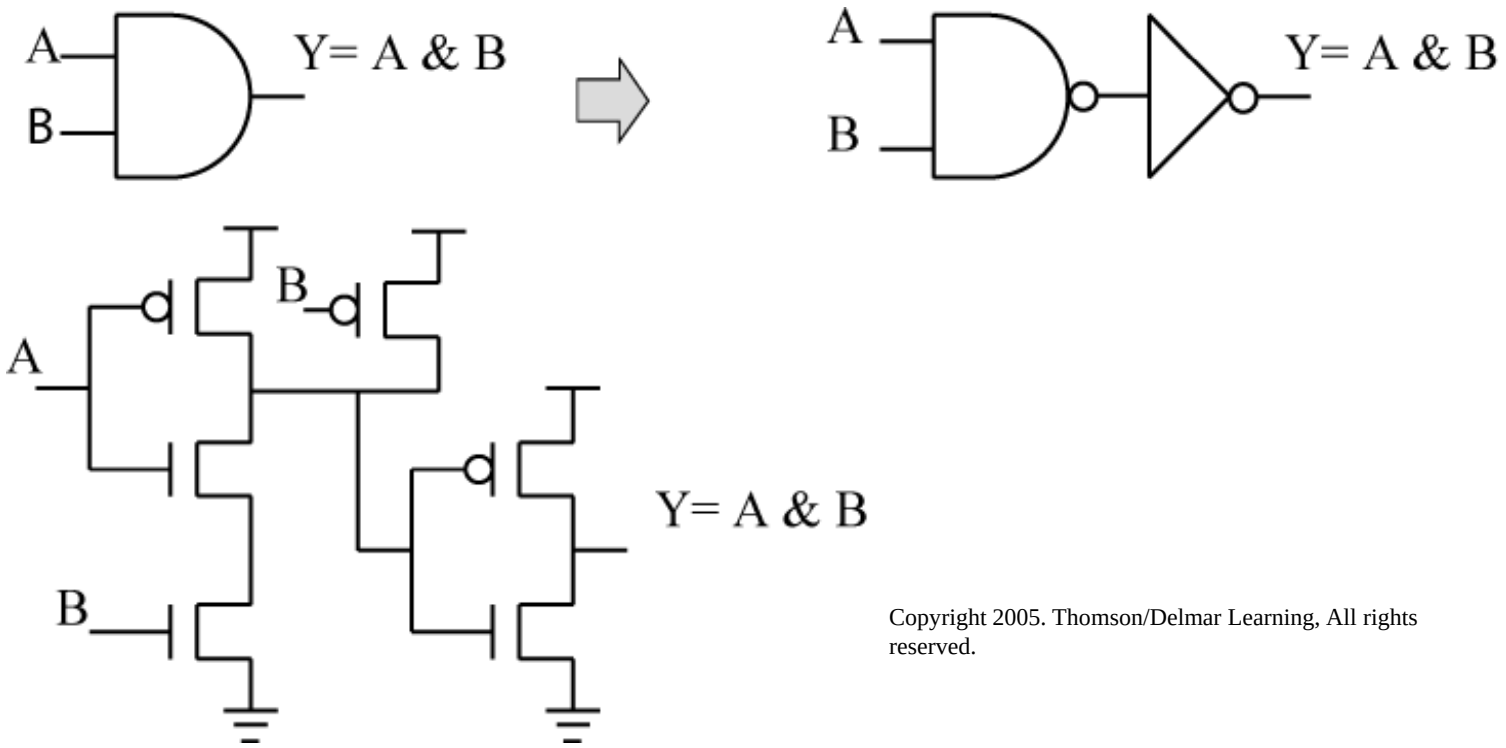
Another logic gate - takes 4 transistors

A B	Y
0 0	
0 1	
1 0	
1 1	



How do we make an AND gate?

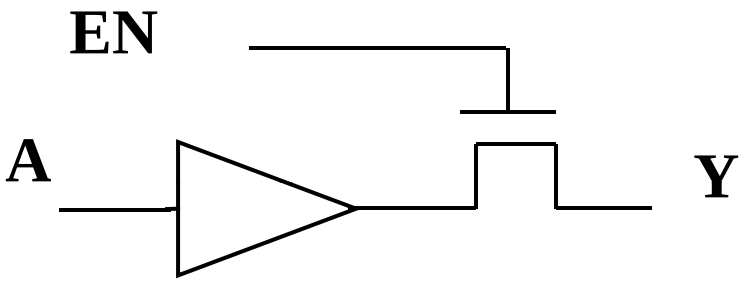
The only way with CMOS transistors is to connect an inverter after a NAND gate.

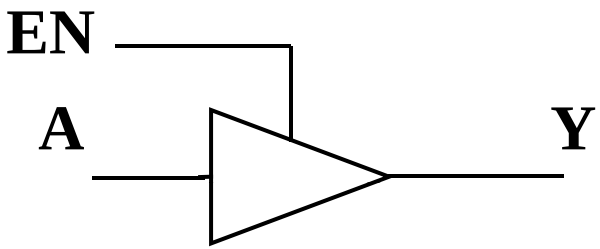


Takes 6 transistors! In CMOS technology, NAND gates are preferable to AND gates because they take less transistors, are faster, and consume less power.

Tri-State Buffer

There is another way to drive a line or bus from multiple sources. Use a TRISTATE buffer.





When EN = 1, then Y = A.

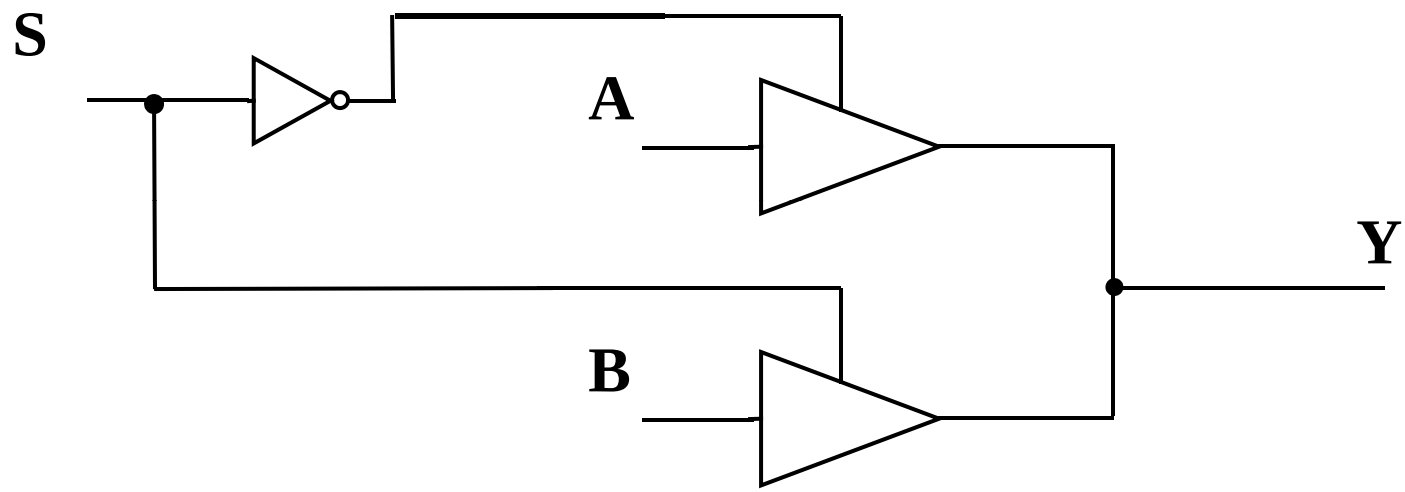
When EN = 0, then Y = ??????
Y is *undriven*, this is called the high impedance state.

Designate high impedance by a 'Z'.

When EN = 0, then Y = 'Z' (high impedance)

Using Tri-State Buffers (cont)

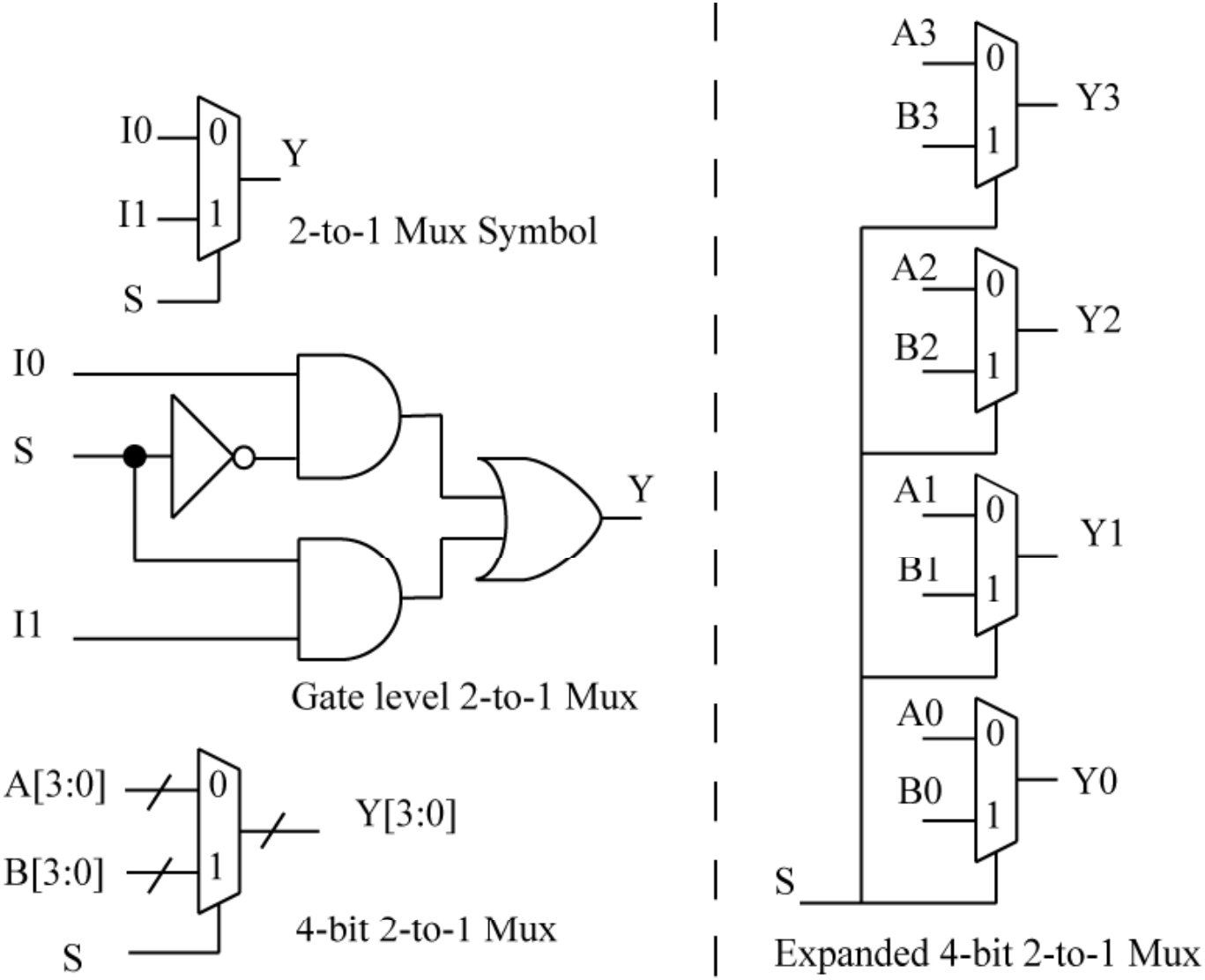
Only A or B is enabled at a time.



Implements 2/1 Mux function

If S=0 then Y = A
If S=1 then Y = B

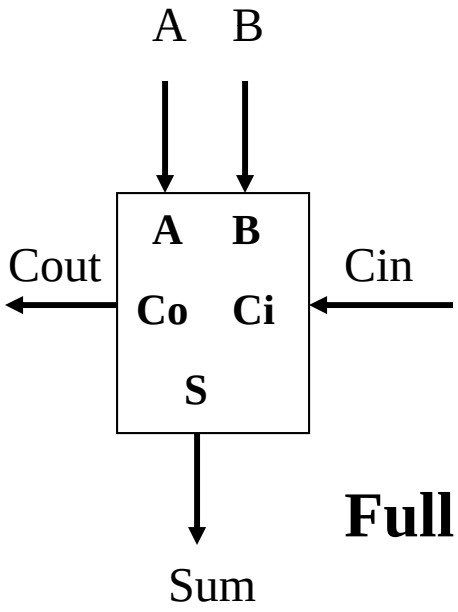
Combinational Building Blocks, Mux



Binary Adder

F (A,B,C) = A xor B xor C **G = AB + AC + BC**

These equations look familiar. These define a *Binary Full Adder* :

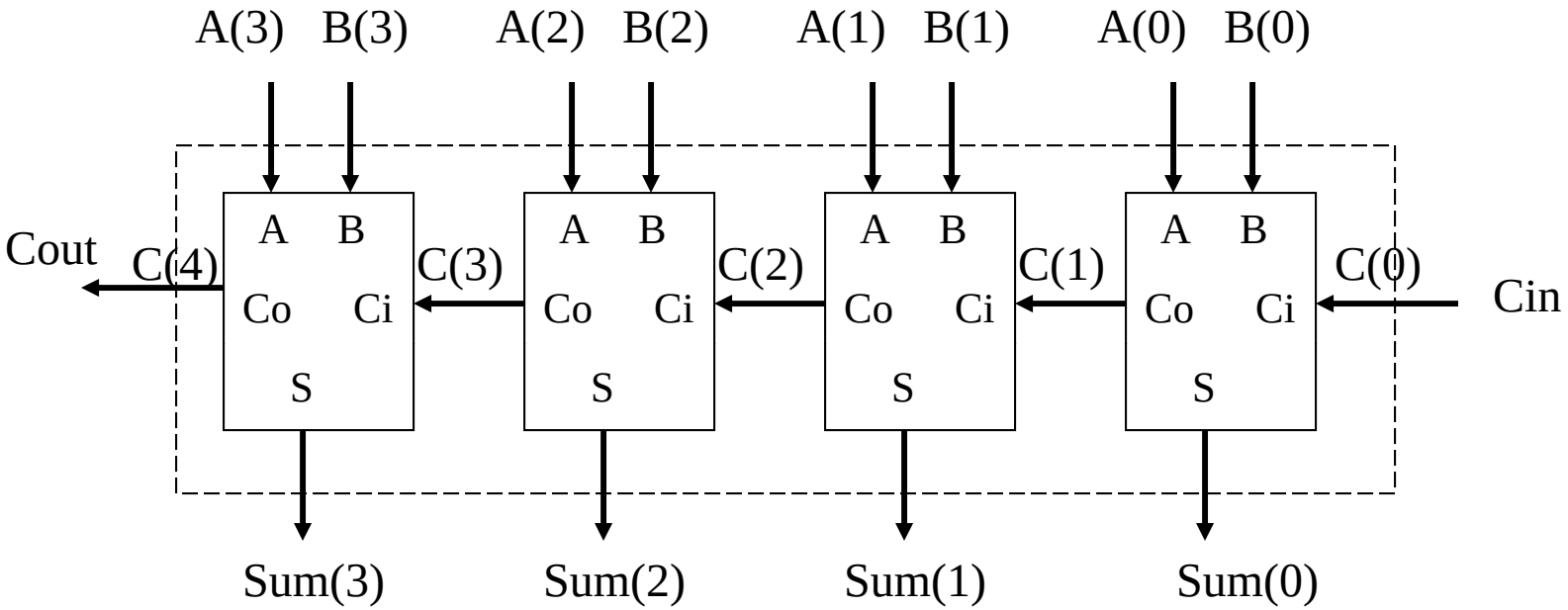


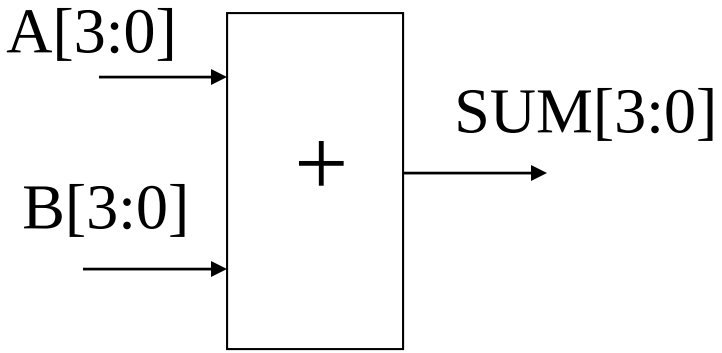
Sum = A xor B xor Cin

Cout = AB + Cin A + Cin B
= AB + Cin (A + B)

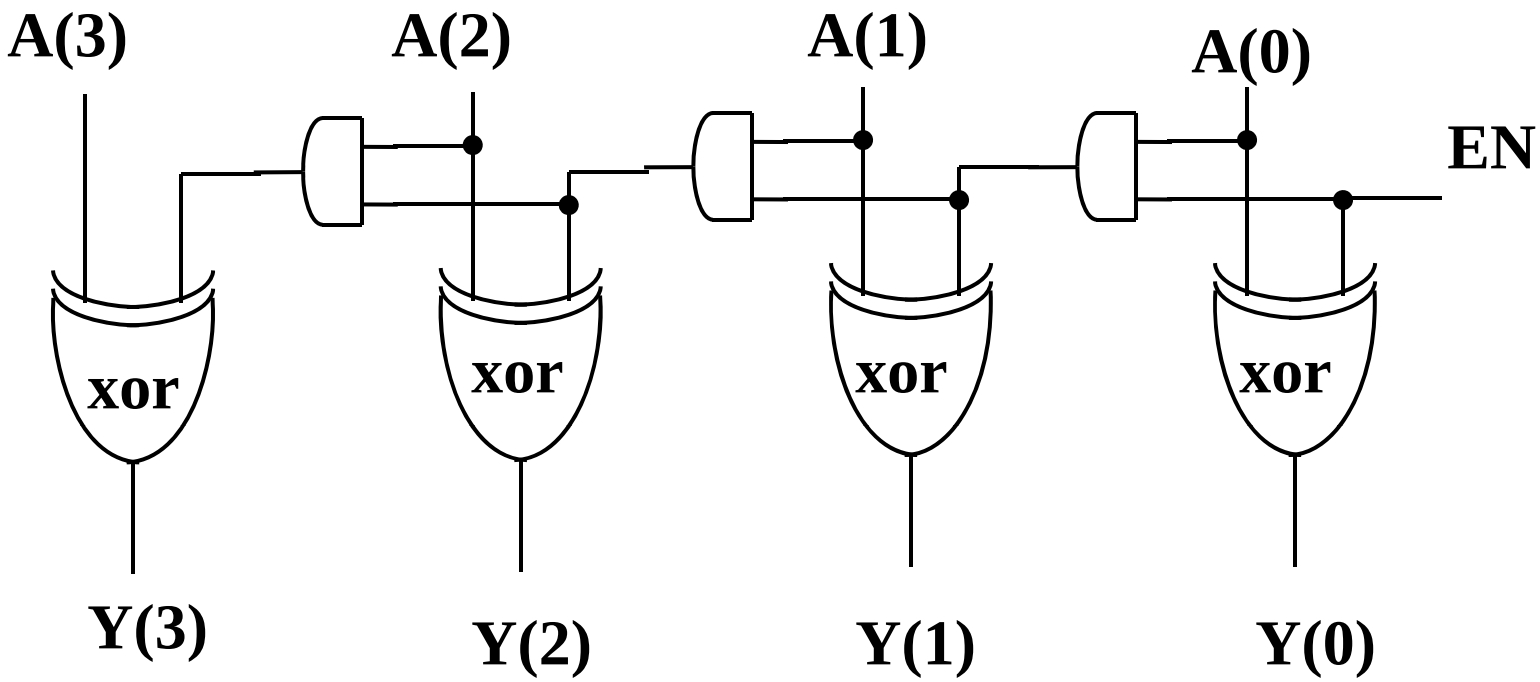
Full Adder (FA)

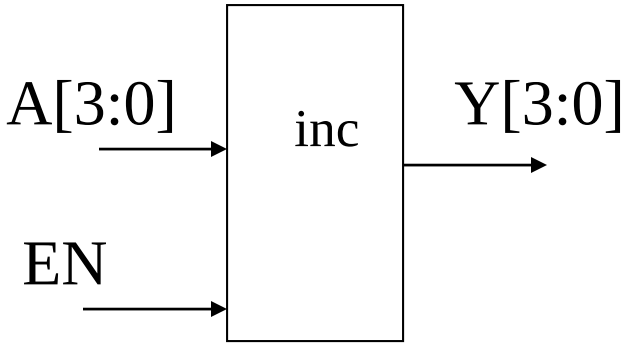
4 Bit Ripple Carry Adder





Incrementer

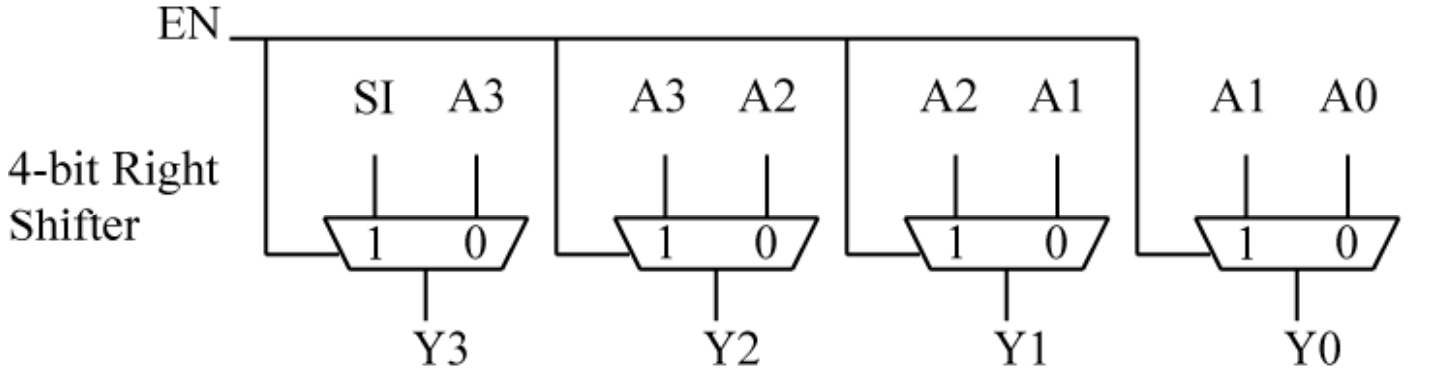


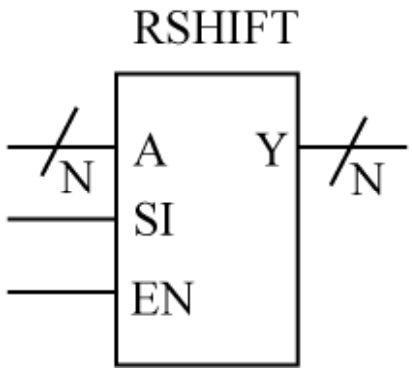


If EN = 1 then Y = A + 1
If EN = 0 then Y = A

Combinational Right Shifter

A combinational block that can either shift right or pass data unchanged



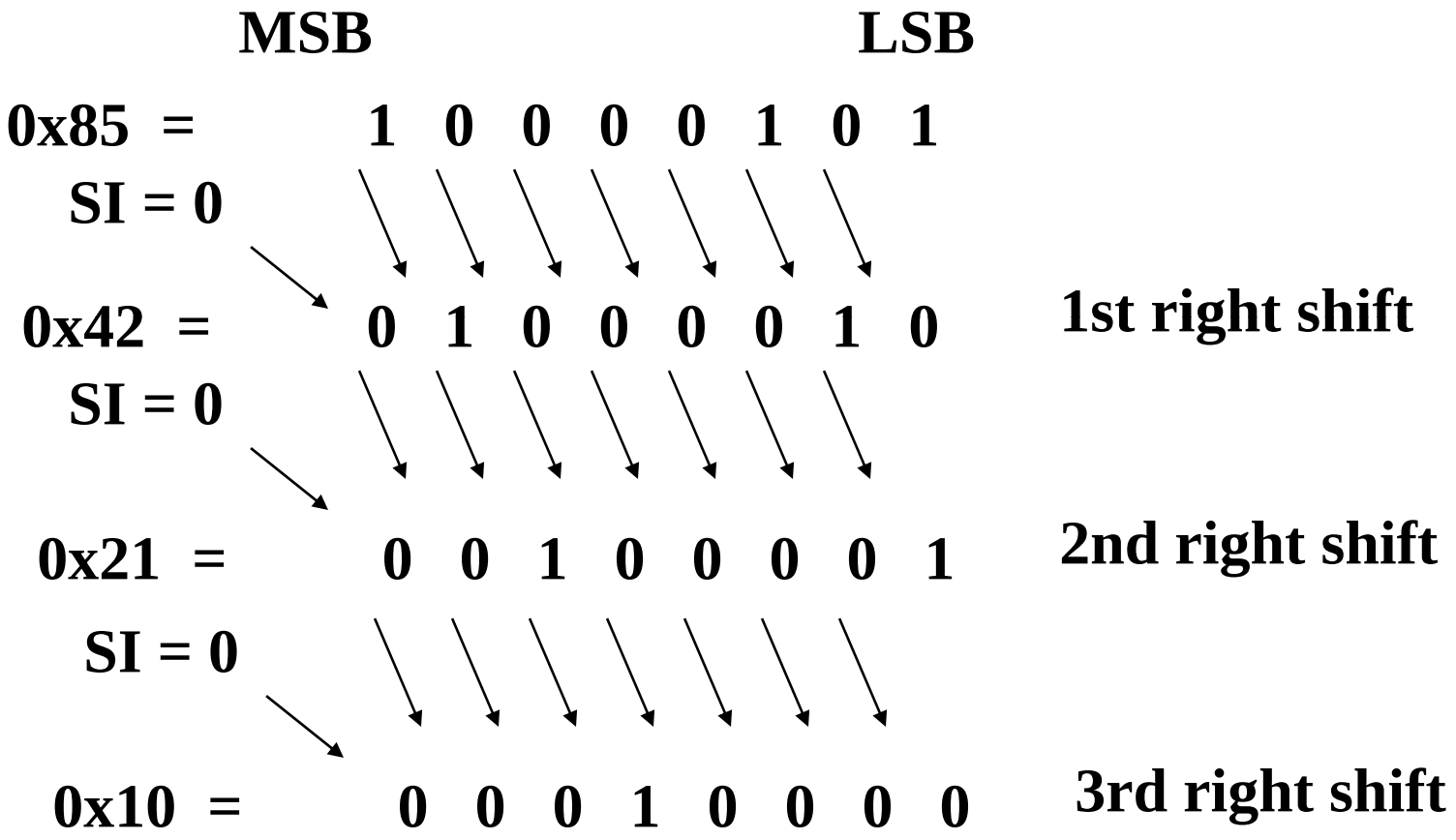


Symbol for N-bit Right Shifter

If EN = 1 then Y = A >> 1

If EN = 0 then Y = A

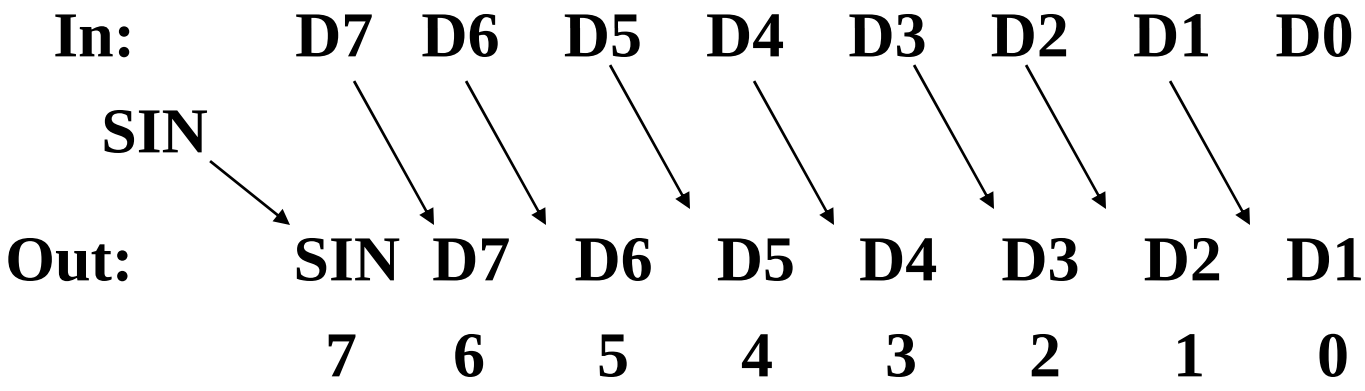
Understanding the shift operation



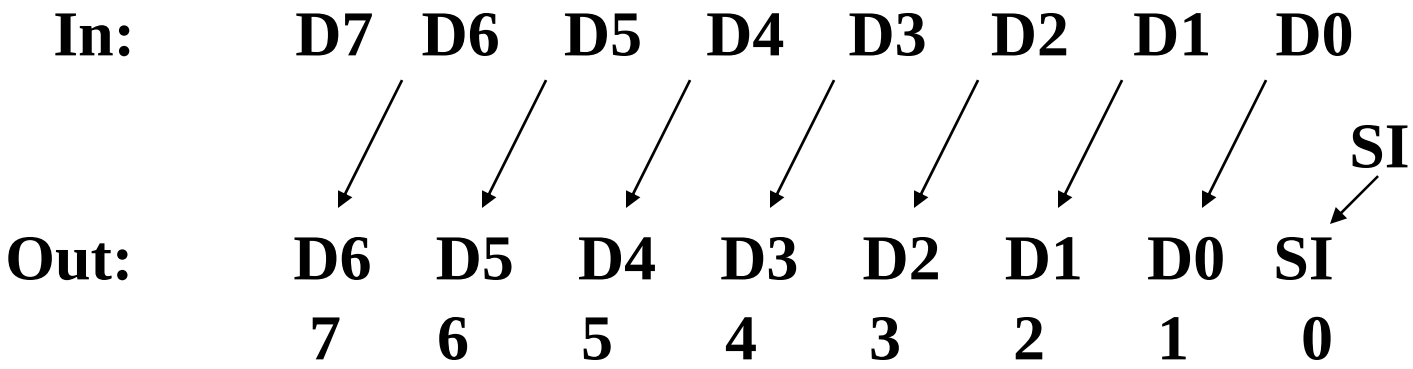
Etc….

Right Shift vs. Left Shift

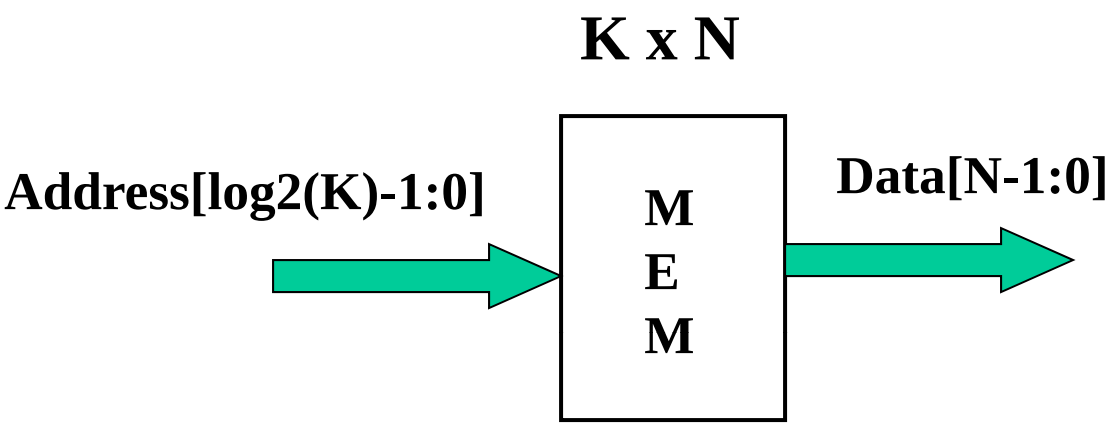
A right shift is MSB to LSB (divide by 2)



A left shift is LSB to MSB (multiply by 2)



Recall Basic Memory Definition



Example:

16 x 8

(16 locations requires log2(16) = 4 address lines, each location stores 8 bits.

Address bus: A[3:0]

Data bus: D[7:0]

K locations, N bits per location

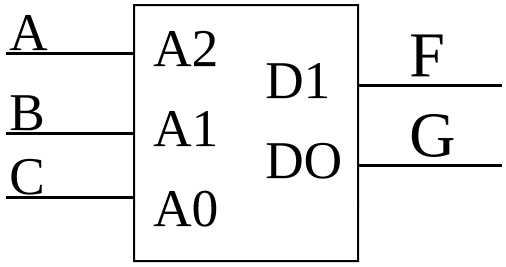
Address bus has log2(K) address lines, data bus has N data lines.

Memory: Implement Logic or Store Data

F (A,B,C) = A xor B xor C G = AB + AC + BC

A	B	C	F	G
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

8 x 2 Memory

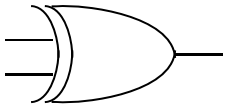


LookUp Table (LUT)

A[2:0] is 3 bit address bus, D[1:0] is 2 bit output bus.
Location 0 has “00”,
Location 1 has “10”,
Location 2 has “10”,
etc….

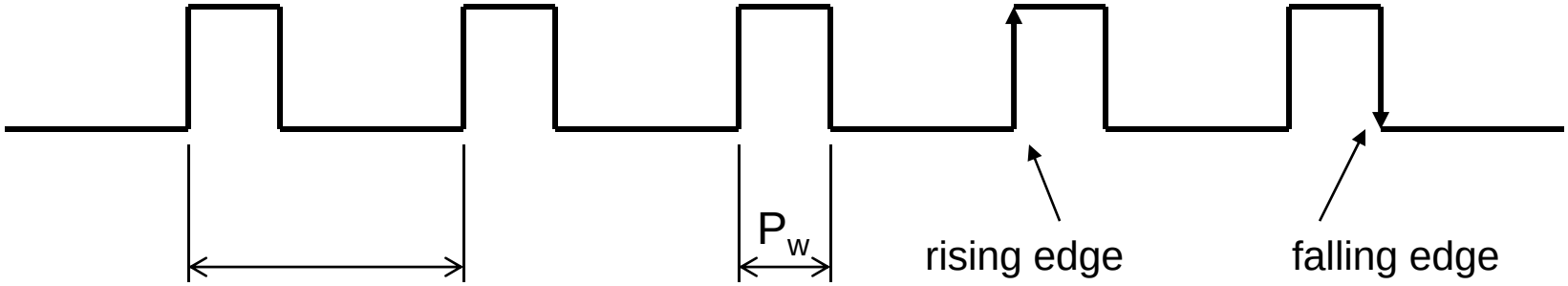
Recall that Exclusive OR (xor) is

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0



Y = A⊕B
= A xor B

Clock Signal Review



voltage

time

τ - period (in seconds)

P_w - pulse width (in seconds)

f - frequency pulse width (in Hertz)

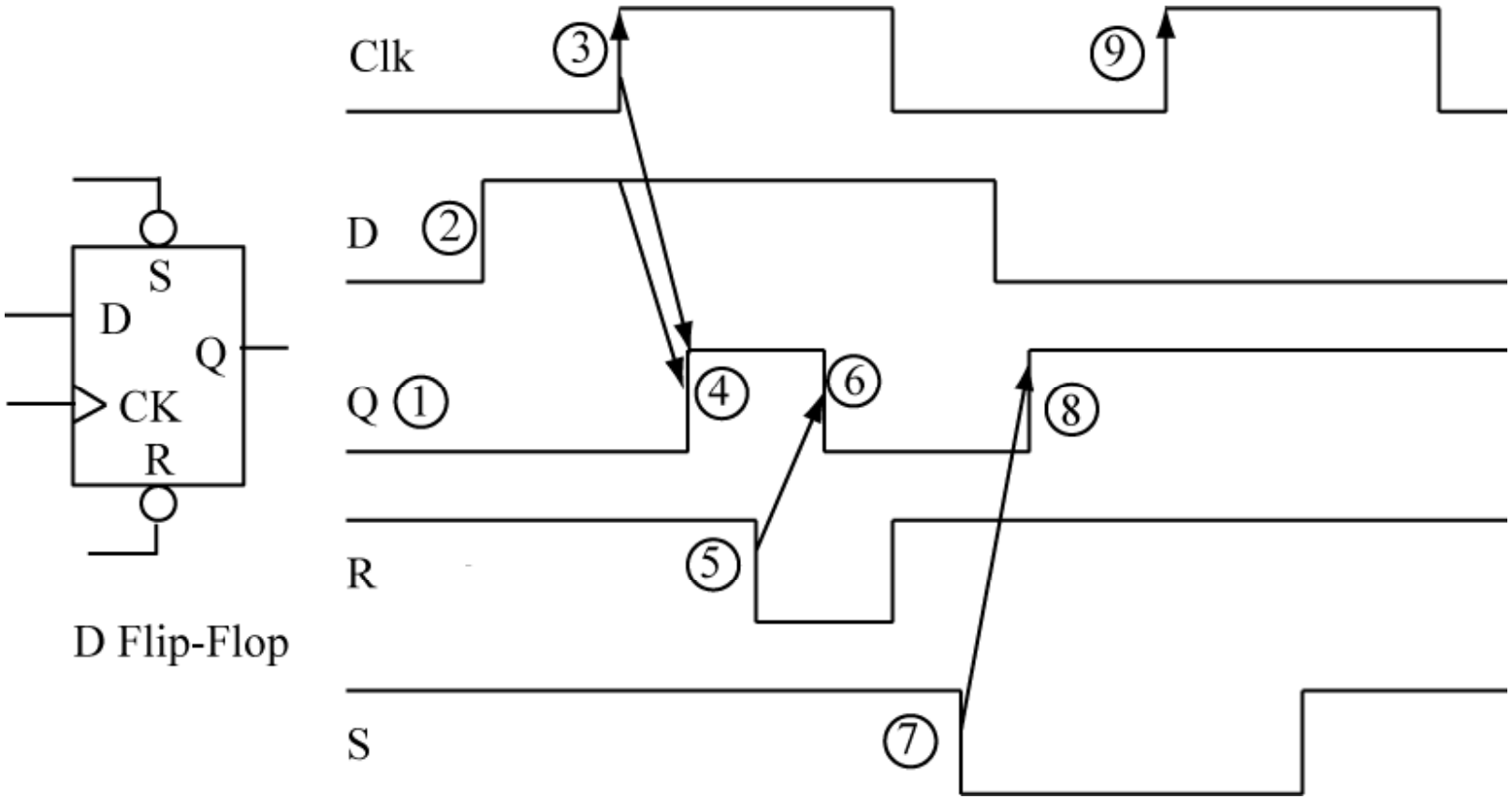
$f = 1/\tau$

duty cycle - ratio of pulse width to period (in %)

duty cycle = P_w / τ

millisecond (ms) 10^{-3}	Kilohertz (KHz) 10^{3}
microsecond (μs) 10^{-6}	Megahertz (MHz) 10^{6}
nanosecond (ns) 10^{-9}	Gigahertz (GHz) 10^{9}

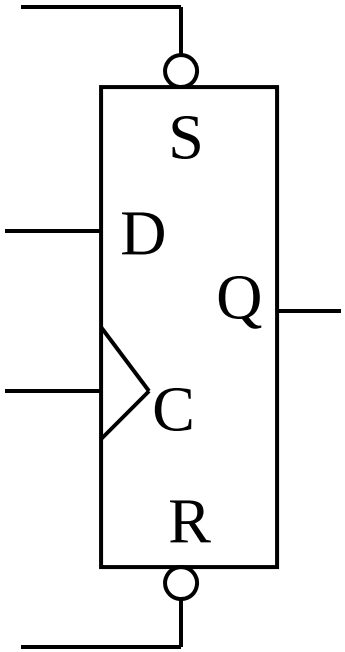
Storage Element: The D Flip-Flop



D: data input
CK: clock input
S : set input (asynchronous, low true)
R: reset input (asynchronous, low true)

Synchronous vs Asynchronous Inputs

Synchronous input: Output will change after active clock edge
Asychronous input: Output changes independent of clock



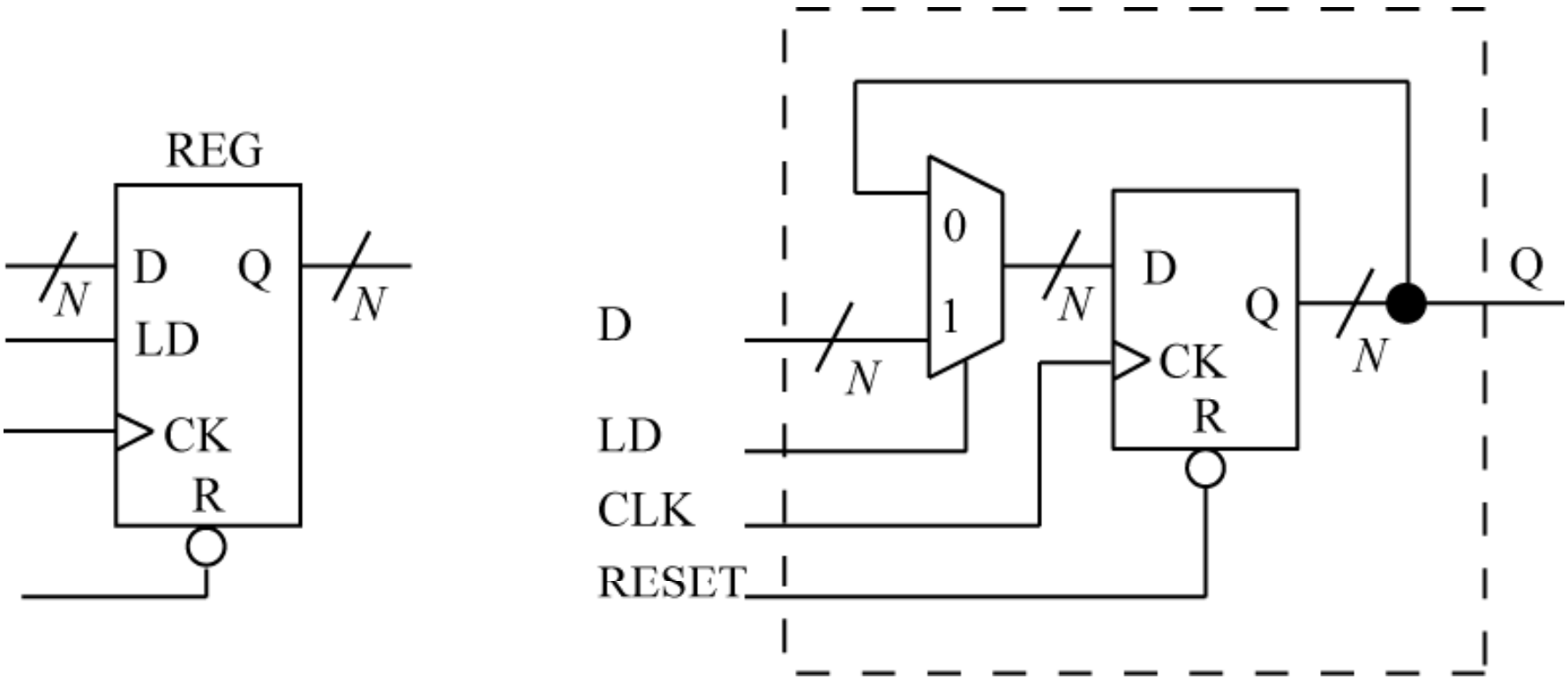
State elements often have async set, reset control.

D input is synchronous with respect to Clk

S, R are asynchronous. Q output affected by S, R independent of C. Async inputs are dominant over Clk.

Registers

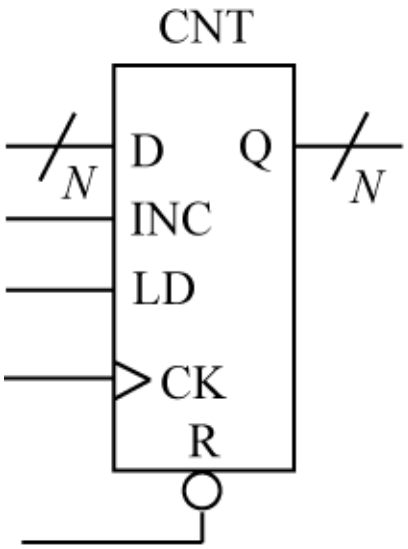
The most common sequential building block is the register. A register is *N* bits wide and has a load line for loading in a new value into the register.

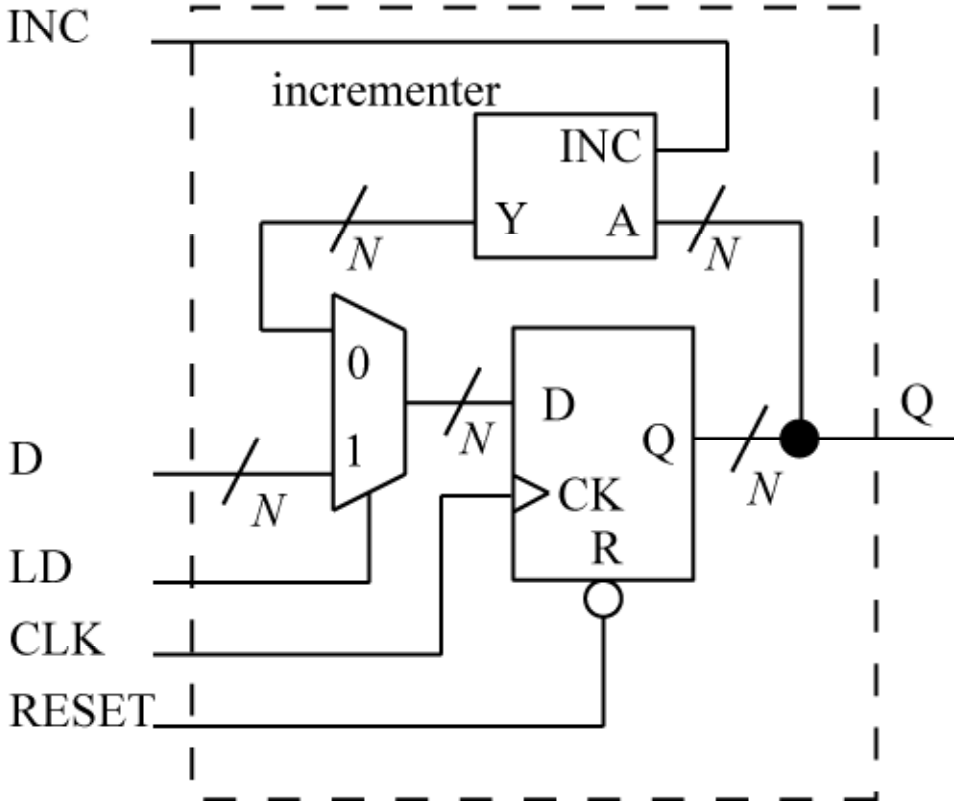


Note that DFF simply loads old value when LD = 0. DFF is loaded every clock cycle.

Counter

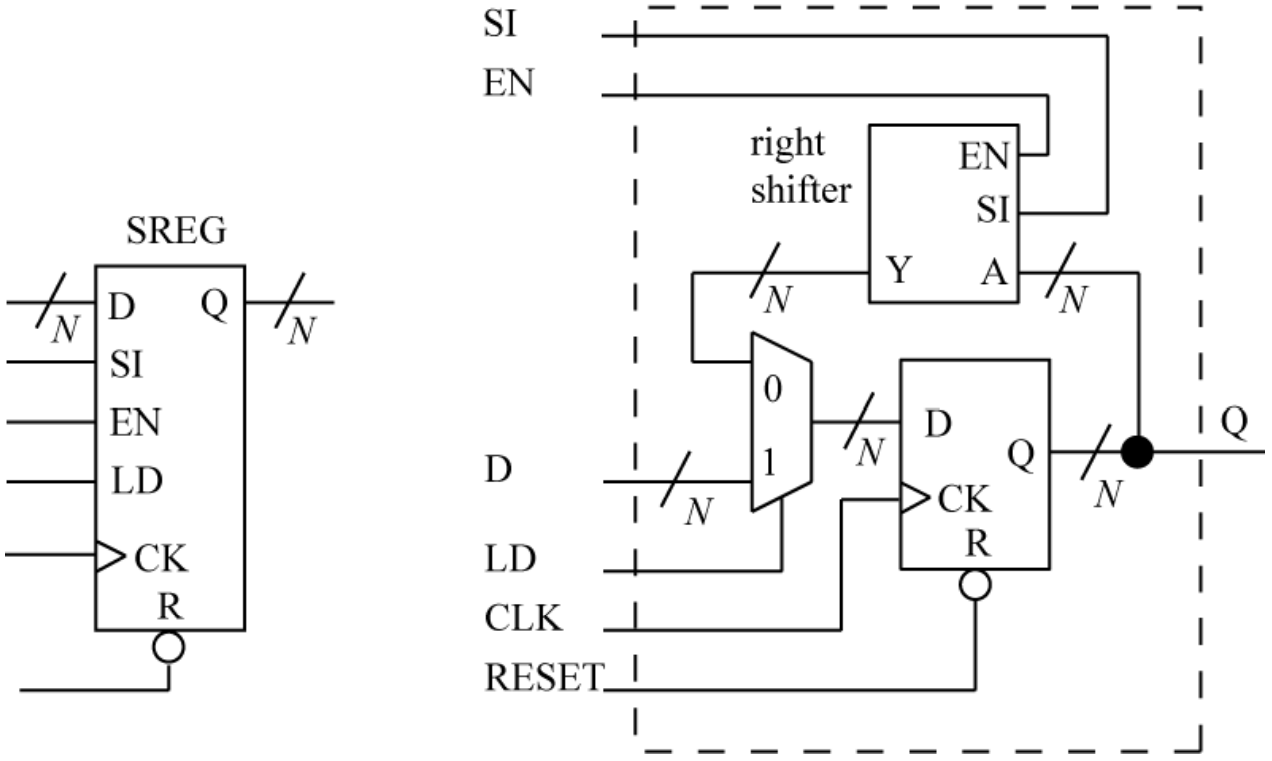
Very useful sequential building block. Used to generate memory addresses, or keep track of the number of times a datapath operation is performed.





Shift Register

Very useful sequential building block. Used to perform either parallel to serial data conversion or serial to parallel data conversion.



Computer = Sequential+ Combinational building blocks/logic

- The next chapter will discuss using combinational + sequential building blocks to build a computer
  - Combinational logic
  - Memory
  - Register
  - Counter

What do you need to Know?

- Convert hex, binary integers to Decimal
- Convert decimal integers to hex, binary
- Convert hex to binary, binary to Hex
- N binary digits can represent 2^N values, unsigned integers 0 to 2^N-1.
- Addition, subtraction of binary, hex numbers
- Detecting unsigned overflow

What do you need to know? (cont)

- ASCII, UNICODE are binary codes for character data
- Basic two-input Logic Gate operation
- NMOS/PMOS Transistor Operations
- Inverter/NAND transistor configurations
- Tri-state buffer operation
- Mux, Memory, Adder operation
- Clock signal definition
- DFF, Register, Counter, Shifter register operation