SDR FRAMEWORK FOR BURST/ CONTINUOUS MPSK/ 16-QAM MODEMS

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ABSTRACT

We describe a software-defined radio (SDR) framework for MPSK/16-QAM pulse-shaping, demodulation, carrier acquisition and tracking for satellite modems. Analytical performance bounds for burst and continuous mode acquisition are validated with measured performance.

1. INTRODUCTION

As satellite launch costs decline, both power (E_b/N_0) and bandwidth efficiency (bps/Hz) determine a transponder's economic value (rather than simply the former). Power efficiency is achieved with lower order modulations (such as BPSK and QPSK), while spectral efficiency is achieved using higher order modulations (HOMs), typically 8-PSK and 16-QAM. Amongst HOMs, constant envelope (CEM) minimizes spectral re-growth modulation associated with high efficiency (e.g., class-C) amplification. For burst-mode multiple-access, reduced preambles (for frequency offset and symbol-timing acquisition) enhance throughput. A satellite transceiver (Figure 1) is generally realized using a configurable software radio [1] with reusable kernels for:

- reduced ISI and spectral re-growth pulse-shaping for all modulations, including HOMs
- burst and continuous mode carrier acquisition
- demodulator algorithms



Figure 1. BPSK, QPSK, 8-PSK and 16-QAM transceiver

In a similar vein, coding standards for 8-PSK and 16-QAM (e.g., for satcom) use pragmatic trellis (PTCM) and outer codes that rely on two "industry standard" kernels: a constraint length 7, rate $\frac{1}{2}$, inner code and Reed-Solomon outer coding over GF(2⁸). Figure 2 shows 8-PSK and 16-QAM PTCM [2] encoders respectively.



Figure 2. Reusable modules (shaded) in ambiguityencoded 8-PSK and $n \cdot \pi/2$ -variant 16-QAM PTCM

2. PULSE-SHAPING

Feher [3] *n*-sample pulse-shapes are extendable to longer window-lengths [4] and HOMs, e.g., for *n*=2, samples for the *k*th 8-PSK or 16-QAM symbol, with bits $[X_kY_kZ_k]$ or $[W_kX_kY_kZ_k]$ respectively, are obtained by indexing Table 1 using 4-bit patterns, $[(X_k \oplus Y_k)Z_k(X_{k+1} \oplus Y_{k+1})Z_{k+1}]$ or $[W_k(W_k \oplus Z_k)W_{k+1}(W_{k+1} \oplus Z_{k+1})]$, for in-phase (*I*), $[X_k\overline{Z}_kX_{k+1}\overline{Z}_{k+1}]$ or $[X_k(X_k \oplus Y_k)X_{k+1}(X_{k+1} \oplus Y_{k+1})]$, for quadrature (*Q*) channels, derived from a 2-symbol input data window $[X_kY_kZ_kX_{k+1}Y_{k+1}Z_{k+1}]$ or $[W_kX_kY_kZ_kW_{k+1}X_{k+1}Y_{k+1}Z_{k+1}]$ (Appendix 1 derives the 8-PSK and 16-QAM PSD for any given pulse-shape set).

$I : X_k \oplus Y_k$	Z_k	$X_{k+1} \oplus Y_{k+1}$	Z_{k+1}	
$I_{\text{QAM}}: W_k$	$W_k \oplus Z_k$	W_{k+1}	$W_{k+1} \oplus Z_{k+1}$	2-sample
$Q : X_k$	\overline{Z}_k	X_{k+1}	\overline{Z}_{k+1}	waveform
$Q_{\text{QAM}}: X_k$	$X_k \oplus Y_k$	X_{k+1}	$X_{k+1} \oplus Y_{k+1}$	
0	0	0	0	<i>a</i> , <i>a</i>
0	0	0	1	<i>z</i> , <i>y</i>
0	0	1	0	Z, -Z
0	0	1	1	<i>z</i> , <i>-x</i>
0	1	0	0	y, z
0	1	0	1	<i>b</i> , <i>b</i>
0	1	1	0	x, -z
0	1	1	1	x, -x
1	0	0	0	-Z, Z
1	0	0	1	- <i>z</i> , <i>x</i>
1	0	1	0	-a, -a
1	0	1	1	-z, -y
1	1	0	0	-x, z
1	1	0	1	-x, x
1	1	1	0	-y, -z
1	1	1	1	-b, -b

Table 1. 16-QAM/ 8-PSK/ QPSK pulse-shaping waveforms with inverse sinc correction (*a*=1, *b*=0.4142, *x*=0.3292, *y*=0.6193, *z*=0.7949) For QPSK, 2-input bits I_k and Q_k , generate the 3-bit symbol subset $[X_k Y_k Z_k]$ of Figure 3 ($X_k = \overline{Q}_k$, $Y_k = I_k \oplus Q_k$, $Z_k = Y_k$; with $I_k = Q_k$ for BPSK) that is used identically to obtain the *k*th symbol's pulse shapes. Thus, a common pulse-shaping ROM is used for MPSK and 16-QAM.



Figure 3. Subset of 16-QAM ($W_k X_k Y_k Z_k$) used for 8-PSK ($X_k Y_k Z_k$), QPSK and BPSK

3. DEMODULATION

Demodulation (Figure 4) comprises efficient I/Q baseband conversion using interpolated FIR (IFIR) decimation filters [5], numerically controlled oscillator (NCO)/ mixerbased frequency-offset tracking, matched filtering and synchronization processing. The latter consists of phase estimation at each baud time offset and using the phase at a selected time offset for coherent demodulation.



Figure 4. Reusable (shaded blocks) demodulator structure

Phase estimation accuracy, obtained at the demodulator frequency tracking range's expense, is determined by the decision-directed phase-locked loop (DDPLL) bandwidth as described in [6] and [7], assuming that frequency jitter, determined by the NCO's sine table length and its phase accumulator's ultimate (word-size dependent) frequency resolution, is small. For MPSK, the DDPLL forms decision variables on each T_s -second baud's in-phase (R_k) and quadrature (S_k) samples and improves carrier phase estimate over an N-symbol block for each of L equally spaced baud time hypotheses¹, σ . For QPSK and BPSK, the phase estimator's decision variables are:

$$T_{rk}(\sigma) = R_k(\sigma)\cos[\theta(\sigma)] + S_k(\sigma)\sin[\theta(\sigma)]], \sigma = 1, \dots, L^{(1)}$$

$$T_{sk}(\sigma) = S_k(\sigma)\cos[\theta(\sigma)] - R_k(\sigma)\sin[\theta(\sigma)]], \sigma = 1, \dots, L^{(1)}$$

For QPSK, hard decoding of $T_{rk}(\sigma)$ and $T_{sk}(\sigma)$ yields channel bits $I_k(\sigma)$ and $Q_k(\sigma)$, which are processed to estimate phase² for each time offset using:

$$R_{sum}(\sigma) = \sum_{N} I_{k}(\sigma) T_{rk}(\sigma) + Q_{k}(\sigma) T_{sk}(\sigma)$$

$$S_{sum}(\sigma) = \sum_{N} I_{k}(\sigma) T_{sk}(\sigma) - Q_{k}(\sigma) T_{rk}(\sigma)$$

$$\theta(\sigma) = \tan^{-1} [S_{sum}(\sigma)/R_{sum}(\sigma)]$$

$$(2)$$

For BPSK, hard decoding of $T_{rk}(\sigma) + T_{sk}(\sigma)$ is used instead: $R_{mm}(\sigma) = \sum I_k(\sigma) [T_{k}(\sigma) + T_{rk}(\sigma)]$

$$S_{sum}(\sigma) = \sum_{N} I_{k}(\sigma) [T_{sk}(\sigma) - T_{rk}(\sigma)], \sigma = 1, \cdots, L$$

$$\theta(\sigma) = \tan^{-1} [S_{sum}(\sigma)/R_{sum}(\sigma)]$$
(3)

An additional step of doubling QPSK decision variables' phase allows the QPSK process to be extended for 8-PSK (and, indeed, to any 2^m -order CEMs, $m \ge 4$) as follows:

Phase doubling:
$$T_{rk}^{'}(\sigma) = T_{rk}^{2}(\sigma) - T_{sk}^{2}(\sigma)$$

 $T_{sk}^{'}(\sigma) = 2 \cdot T_{rk}(\sigma) \cdot T_{sk}(\sigma)$, $\sigma = 1, \dots, L$ (4)

Hard decoding of $T'_{rk}(\sigma)$ and $T'_{sk}(\sigma)$ yield decisions $I_k(\sigma)$ and $Q_k(\sigma)$, which are then used by the phase estimator (for each time offset) as follows:

$$R_{sum}(\sigma) = \sum_{N} I_{k}(\sigma) T_{rk}(\sigma) + Q_{k}(\sigma) T_{sk}(\sigma)$$

$$S_{sum}(\sigma) = \sum_{N} I_{k}(\sigma) T_{sk}(\sigma) - Q_{k}(\sigma) T_{rk}(\sigma)$$

$$\theta(\sigma) = 0.5 \tan^{-1} [S_{sum}(\sigma)/R_{sum}(\sigma)]$$
(5)

The 8-PSK PTCM decoder inputs are quantized phase (ambiguous to 180°) rotated decision variables.

For Gaussian noise and equi-probable 16-QAM, phase (ambiguous to $n \cdot \pi/2$, n=1,2,3) is calculated from any one channel's (e.g., Q) 4th moment to squared variance ratio³:

$$\phi = \frac{\pi}{8} - \frac{1}{4} \cdot \sin^{-1} \left[\frac{1}{b} \cdot \left(a \left\{ 1 + \frac{1}{3.667} \cdot \frac{N}{S} \right\} - \frac{E[Q]^4]}{E[Q]^2} \right] \right]$$
(6)

Since $E_s/N_0>14dB$ for 16-QAM, signal-to-noise ratio estimation in (6) may be omitted. *a* and *b*, 1.98 and 0.34 respectively for unshaped 16-QAM, weakly depend on pulse-shaping. Near $\phi=\pi/8$, the arcsin function may be omitted, yielding Figure 5's 16-QAM DPLL structure. AGC-based pre-scaling (shown in Figure 4) aids fixedpoint calculation of $E[Q^4]/E^2[Q^2]$. The Viterbi decoder resolves $\pm 90^\circ$ ambiguity, while the unique-word based R-S frame synchronizer resolves 180° ambiguity.

¹For time offsets, $\sigma=1, ..., L$, the symbol synchronized offset selected, σ_m , has the maximum of all block energies $E(\sigma) = R_{sum}^2(\sigma) + S_{sum}^2(\sigma)$. Parallel demodulation of T_s/L -spaced time offsets (rather than sequential timing recovery) allows shortening of burst-mode preamble, usually composed of un-modulated carrier recovery (CR), bit-timing recovery (BTR) and low aperiodic autocorrelation unique word (UW), by omitting BTR.

²The signs of *y* and *x* select an unambiguous quadrant of $\tan^{-1}(y/x)$.

³Determining the hard decoding signal set to realize a DDPLL for a nonlinearly amplified 16-QAM increases computational requirements.



Figure 5. Kurtosis-based 16-QAM digital PLL (modules common to MPSK DDPLL are shaded); *n*>*k*

QPSK, 8-PSK and 16-QAM soft decisions are $T_{rk}(\sigma_m)$ and $T_{sk}(\sigma_m)$, BPSK using just $T_{rk}(\sigma_m)+T_{sk}(\sigma_m)$. E_b/N_0 vs. BER for QPSK, 8-PSK (TCM metrics set in [8]) and 16-QAM (PLL phase noise included) are shown in Figure 6.



Figure 6. QPSK/ 8-PSK/16-QAM BER performance

During burst-mode frequency and phase⁴ acquisition, the BPSK/ QPSK DDPLL yields an unambiguous carrier phase with respect to each assumed baud time hypothesis. Using the QPSK DDPLL for 8-PSK burst-mode⁵ acquisition circumvents the 8-PSK phase estimator's inherent 180° phase ambiguity, thereby yielding a frequency tracking range $f_t = k_1/(4 \cdot T_s \cdot M \cdot N)$ Hz (k_1^{-1} =phase tracking time constant) for MPSK. Similarly, the QPSK DDPLL is also used for 16-QAM burst-mode acquisition.

4. CARRIER ACQUISITION

For continuous mode MPSK, carrier acquisition removes modulation (exponentiating⁶ the signal, and thereby reducing in-band S/N, by M), while for burst-mode, an unmodulated preamble field is exploited for quick acquisition. In both cases, the receiver applies a

Chebyshev window⁷ and calculates DFT⁸ envelopes, y_1 and y_2 , at two equidistant bins from the nominal carrier. Averaging periodograms over *K* (4 for burst mode and 1024 for continuous mode) 50% overlapped segments improves SNR by almost *K*. The frequency offset, Δf , is obtained as a function of y_2/y_1^9 (shown in Figure 7 for burst-mode), where f_{bin} is the optimum DFT bins' position (minimizing the probability that frequency offset error, over a desired acquisition range and SNR, exceeds the PLL's frequency tracking range) from the nominal carrier, normalized to the DFT window duration's reciprocal.



Figure 7. Relation between frequency offset and y_2/y_1

The continuous mode functional relationship depends, due to pulseshaping- and ISI- induced spectral envelope fluctuations, upon modulation type, window length and baud-rate as shown by Table 2 and Figure 8.

Operating Mode	Baud rate (symbols/sec)	Wind duratio	low n (µs)	Side-lobe (attn. dB)	
		Coarse	Fine	Coarse	Fine
		acq.	acq.	acq.	acq.
Burst- mode	1024k	15.625	-	40	-
	512 k	31.25	-	44	-
	256 k	62.5	-	46	-
	128 k	125	-	48	-
Continuous	1024k	5.2	10.4	33	40
	512 k	10.4	20.8	40	43
DDCV	256 k	20.8	41.6	43	44
BPSK	128 k	41.6	83.2	44	45
Continuous mode QPSK	1024k	5.2	10.4	38	45
	512 k	10.4	20.8	45	52
	256 k	20.8	41.6	52	54
	128 k	41.6	83.2	54	57
Continuous	1024k	5.2	10.4	27	40
Continuous	512 k	10.4	20.8	40	53
	256 k	20.8	41.6	53	54
8-P5K	128 k	41.6	83.2	54	59
Continuous	1024k	10.4	20.8	40	52
mode 16-QAM	512 k	20.8	41.6	43	57
	256 k	41.6	83.2	41	58
	128 k	83.2	166.4	44	62

Table 2. Chebyshev acquisition windows

⁴Absolute encoding provides improved burst-mode BER performance with only slightly increased burst loss probability due to cycle slips.

⁵Continuous-mode decoding (inverting Figure 2's ambiguity encoder) recovers data for $k\pi/8$, k=0,...,7, initial phase offsets or cycle-slips.

⁶By 16 for 16-QAM, since its phases are approximately a subset of 12 of 16 equally-spaced phases. This approximation, as well as amplitude modulation, results in a less localized spectral peak after exponentiation, which Figure 8's 16-QAM offset curves take into account.

⁷Chebyshev windows [9], with $1.5 \le \alpha \le 3$, suppress out-of-band noise and interference by 30-60dB, while incurring processing losses of 3-3.23dB.

⁸ DFT-based acquisition is robust in many noise/ interference scenarios.

⁹This estimator enables an acquisition procedure which decreases the offset progressively and rapidly; in contrast, a PLL's phase estimates become uncorrelated once the offset exceeds its limited tracking range.



Figure 8. Relationship between offset and y_2/y_1 for the continuous modes of Table 2

The burst loss probability, at offset f_0 and $M(\cdot)$ mapping Δf to r, is:

$$p\left(\left|f_{0} - \hat{f}_{0}\right| > f_{t}\right) = 1 - \int_{M(f_{0} - f_{t})}^{M(f_{0} + f_{t})} f_{\hat{r}}(\hat{r}/f_{0}) \, d\hat{r}$$
(7)

The pdf of the ratio, $\hat{r} = \hat{y}_1/\hat{y}_2$, has upper-bound:

$$f_{\hat{r}}(\hat{r}/\Delta f) \le \sqrt{\frac{\hat{r}}{y_1 y_2}} \cdot \frac{1}{2\pi\sigma^2} \int \hat{y}_2^2 \cdot e^{\frac{-\left[(\hat{r}\hat{y}_2 - y_1)^2 + (\hat{y}_2 - y_2)^2\right]}{2\sigma^2}} d\hat{y}_2, \quad \text{for}$$

 $\frac{y_1 \cdot \hat{y}_1}{\sigma^2} \gg 1$, $\frac{y_2 \cdot \hat{y}_2}{\sigma^2} \gg 1$, where \hat{y}_1 , \hat{y}_2 are estimated DFT

envelopes. Figures 9 and 10 show upper bounds on singleiteration lock failure probabilities for 8-PSK continuous and MPSK burst modes.



Figure 9. Single-iteration 8-PSK/ PTCM continuous mode lock failure probabilities as functions of normalized (to 96kHz) frequency offsets, with f_t =0.0167.



Figure 10. Burst loss probability¹⁰ for different normalized (to 64kHz) frequency offsets. $f_t=0.125$.

The look-up table-based frequency offset estimator has poor discrimination at small offsets¹¹ (normalized to the reciprocal of window duration) and high noise susceptibility at large offsets. In continuous mode, using short windows initially (for large offsets) and progressively longer windows as the offset reduces allows frequency acquisition for both small and large offsets. The longest window used decides the error probability, as the iterative algorithm eventually drives (by tuning a NCO) to lower offsets. Moreover, even with a fixed window length, the probability of acquisition improves at lower offsets, as illustrated by the pdfs of the ratio \hat{y}_2 / \hat{y}_1 in Figure 11.



Figure 11. Reduced variance pdfs at lower offsets allow iterated acquisition in continuous mode

5. CONCLUSION

Re-usable and re-configurable modules that realize multimode modems, e.g., in an SDR framework [10], overcomes memory/ DSP processing power (or FPGA gate-count/ speed) limitations, without compromising performance. We show that:

- Pulse-shaping components used for HOM, i.e., 16-QAM, may be reused for lower order CEMs
- QPSK demodulation modules may be reused for all CEMs and 16-OAM
- A common module may acquire CEM/ 16-QAM carrier.

¹⁰ Network-specific burst acquisition performance is further enhanced, e.g., in a TDMA network, when a receiver initializes its LO using a table of slot frequency offset estimates (that is updated after each successfully received burst), while limiting LO correction during acquisition. ¹¹ In contrast, the DDPLL excels at tracking small offsets.

APPENDIX 1

With m=3 symbols decision window the power spectrum of the pulse-shaped signal is:

$$S(f) = \frac{1}{T_s} \sum_{j=0}^{4^m - 1} \pi_j \left| s_j(f) \right|^2 + \frac{2}{T_s} \sum_{j=0}^{4^m - 14^m - 1} \pi_j s_j^*(f) s_k(f) Q_{jk}(f), \text{ where } s_j(f)$$

is the *j*th pulse's Fourier transform, π_j the *j*th state's stationary probability (i.e., 1/64), $Q_{jk}(f) = \sum_{n=1}^{\infty} p_{jk}^{(n)} e^{-j2\pi j n Ts}$ the *j*-to-*k* state transition probability generating function, derived from $p_{jk}^{(n)}$, the *j*-to-*k n*-step transition probability.

$$Q_{10} = \frac{1}{64} e^{-j6\pi T_s} + \frac{1}{64} e^{-j8\pi T_s} + \dots = \frac{1}{64} \cdot \frac{e^{-j6\pi T_s}}{1 - e^{-j2\pi T_s}}$$

$$Q_{04} = \frac{1}{16} e^{-j4\pi T_s} + \frac{1}{64} e^{-j6\pi T_s} + \dots = \frac{1}{16} \cdot e^{-j4\pi T_s} + Q_{10}$$

$$Q_{00} = \frac{1}{4} e^{-j2\pi T_s} + \frac{1}{16} e^{-j4\pi T_s} + \frac{1}{64} e^{-j6\pi T_s} + \dots = \frac{1}{4} \cdot e^{-j2\pi T_s} + Q_{04}$$

$$Q_{14} = \frac{1}{4} e^{-j2\pi T_s} + 0 + \frac{1}{64} e^{-j6\pi T_s} + \dots = \frac{1}{4} \cdot e^{-j2\pi T_s} + Q_{10}$$

For Table 1's pulse-shapes, the generating functions are:

	-	0 - 3	4 - 7	8 - 11	12 - 15	16 - 19	20 - 23	24 - 27	28 - 31	
	0	Q_{00}	Q_{04}	Q_{04}	Q_{04}	Q_{10}	Q_{10}	Q_{10}	Q_{10}	
B=	1	Q_{10}	Q_{14}	Q_{10}	Q_{10}	Q_{04}	Q_{04}	Q_{04}	Q_{04}	
	2	Q_{10}	Q_{10}	Q_{14}	Q_{10}	Q_{10}	Q_{10}	Q_{10}	Q_{10}	
	3	Q_{10}	Q_{10}	Q_{10}	Q_{14}	Q_{10}	Q_{10}	Q_{10}	Q_{10}	
	4	Q_{04}	Q_{04}	Q_{04}	Q_{04}	Q_{14}	Q_{10}	Q_{10}	Q_{10}	
	5	Q_{10}	Q_{10}	Q_{10}	Q_{10}	Q_{04}	Q_{14}	Q_{04}	Q_{04}	
	6	Q_{10}	Q_{10}	Q_{10}	Q_{10}	Q_{10}	Q_{10}	Q_{14}	Q_{10}	
	7	Q_{10}	Q_{14}							
	8	Q_{04}	Q_{04}	Q_{04}	Q_{04}	Q_{10}	Q_{10}	Q_{10}	Q_{10}	
	9	Q_{10}	Q_{10}	Q_{10}	Q_{10}	Q_{04}	Q_{04}	Q_{04}	Q_{04}	
	10	Q_{10}								
	11	Q_{10}								
	12	Q_{04}	Q_{04}	Q_{04}	Q_{04}	Q_{10}	Q_{10}	Q_{10}	Q_{10}	
	13	Q_{10}	Q_{10}	Q_{10}	Q_{10}	Q_{04}	Q_{04}	Q_{04}	Q_{04}	
	14	Q_{10}								
	15	Q_{10}								
or matrix				B Fl	3],					

The generator matrix
$$Q = \begin{bmatrix} B & FB \\ B & FB \\ B & FB \end{bmatrix}$$
,
where *FB* is the matrix obtained h

where FB is the matrix obtained by flipping B first leftright and then up-down. Interpolating the 2-sample waveforms (in Table 1) by periodic replication and using an up-sampling filter, the spectrum of each pulse is obtained as:

$$s_j(f) = \frac{1}{N} \sum_{n=-(N-1)/2}^{(N-1)/2} s_j(n) \cdot \exp\left(\frac{-j2\pi f n}{N}\right)$$

The pulse-shaped 8-PSK power-spectrum, obtained using interpolated pulses, with 5.35dB suppression at 0.167 of the uncoded bit rate, is shown in Figure 12. A surface acoustic wave filter further suppresses modulator side-lobes in order to meet the IESS-310 [11] spectral mask¹².



Figure 12. 8-PSK pulse-shaped spectrum

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¹² An equal symbol-rate unequally-spaced (with phase error at intermediate amplitude being $\approx 4^{\circ}$ from ideal) 16-QAM constructed using *I*, $Q=\pm a$ and $\pm b$ (the same levels as 8-PSK), with identical pulse-shaping, has identical spectrum (as the *I/Q* NRZ transitions remain uncorrelated and there being no change to the transition probabilities $p_{ik}^{(n)}$).