AN 012

Accelerometer Errors

Introduction

Although everyone would like them to be, sensors are not perfect. Understanding the accelerometer's errors is just as important as understanding how the accelerometer works in an application. Errors can and will affect the application's performance.

This application note explains the sources of error in Kionix MEMS tri-axis accelerometers. Suggestions are made on how to reduce or eliminate the error in the measurements. In this way, applications can be made more accurate and precise.

Accelerometer calibration

Calibrating accelerometers is mostly a factory test problem. Kionix trims the offset and sensitivity of each accelerometer by adjusting gain and offset trim codes stored in EEPROM. Ultimately, the accuracy of every accelerometer is limited by the bit resolution for the sensitivity and offset. In addition, test tolerances are also placed on the sensor during the programming process. When the programming station adjusts the sensitivity and offset to be within 1% of target, the station considers the calibration complete. Finally, the accuracy of the test station and its calibration influence the accuracy of the accelerometers.

Customers buying accelerometers and assemblying them into products can also have factory test issues in regards to accelerometer calibration. Because there can be assembly issues that result in accelerometers that are rotated or tilted relative to the desired position for the application, customers occasionally want to re-zero the 0g offset or level the pitch and roll in their production line. Some possible techniques for doing this are detailed below. Obviously test costs must be assessed with the desired accuracy and the quality of accelerometers.

Correction of Sensor Bias Error

Sensor bias error, as defined here, is the difference between the ideal 0g output and the 0g output reported by the sensor. Think of a perfectly horizontal surface with a Kionix accelerometer sitting on it. If there were no bias error, then the sensor output would read the ideal 0g offset voltage ($V_{dd}/2$) on the x and y-axis, +1g output voltage on the z-axis. However, the sensor will read something different than the ideal output on a perfectly horizontal surface because of many factors including mechanical tolerances in the component parts (PCB, screws, standoffs, solder pads, etc.).

The simplest way to measure and remove bias error is to do the following:

1. Place the sensor module on a flat horizontal surface (table top, granite flat, etc.).
2. Read the outputs a_{x1}, a_{y1}, and a_{z1}.
3. Calculate the sensor module biases where S_{zz} is the sensitivity of the z-axis sensor in V/g:

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$$B_x^{0g} = a_{x1}; \quad B_y^{0g} = a_{y1}; \quad B_z^{0g} = a_{z1} - S_{zz} * 1g$$

Record B_x^{0g}, B_y^{0g}, and B_z^{0g} in EEPROM or flash memory and subtract it from all subsequent measurements to get the corrected outputs.

Obviously, the most significant error in this technique comes from the flatness of the horizontal surface. If the surface is actually tilted by one or two degrees, then this tilt has been "calibrated" into the accelerometer outputs. When the sensor module is placed on a true flat (0 degree) surface, the outputs will not be accurate. They will be off by one or two degrees.

To correct for the possibility of a tilted surface, do the following:
1. Place the sensor module on an approximately horizontal surface (table top, granite flat, etc.).
2. Read the outputs a_{x1}, a_{y1}, and a_{z1}.
3. Rotate the sensor module 180 degrees on the surface so that it is facing the opposite direction (see Figure 1).
4. Read the outputs a_{x2}, a_{y2}, and a_{z2}.
5. Calculate the sensor module biases:

$$B_x^{0g} = \left(\frac{a_{x1} + a_{x2}}{2}\right); \quad B_y^{0g} = \left(\frac{a_{y1} + a_{y2}}{2}\right); \quad B_z^{0g} = \left(\frac{a_{z1} + a_{z2}}{2}\right) - S_{zz}$$

6. Record B_x^{0g}, B_y^{0g}, and B_z^{0g} in EEPROM or flash memory and subtract it from all subsequent measurements to get the corrected outputs.

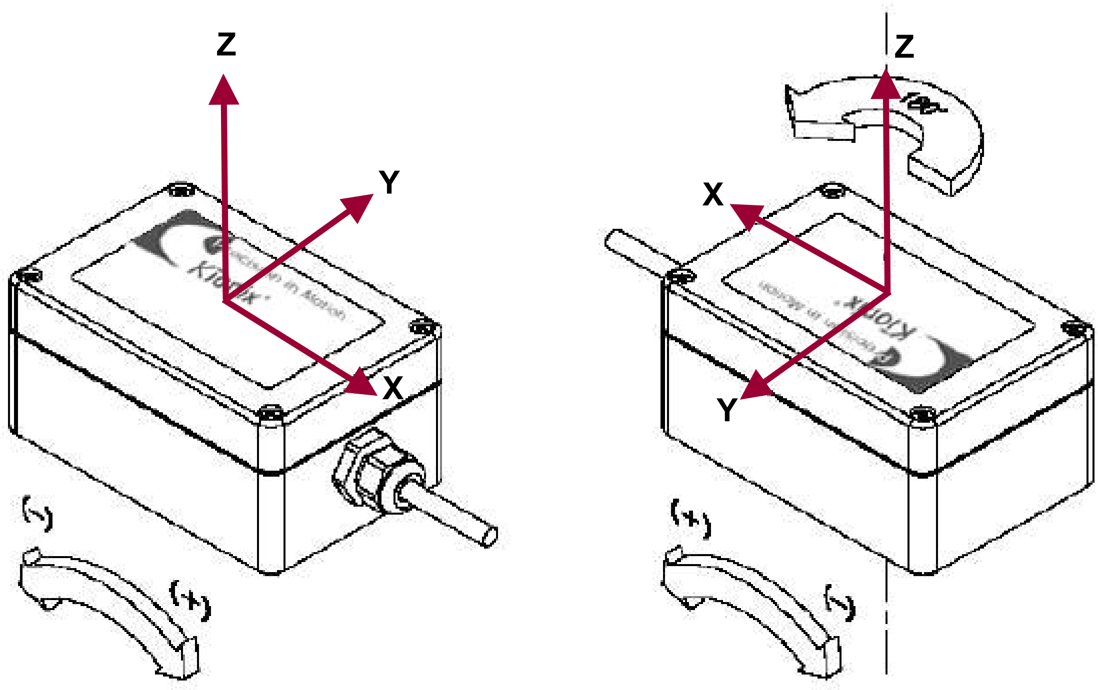


Fig. 1) Rotating a sensor module 180 degrees on a flat surface

Correction of Sensor Bias Error and Sensitivity Error

If more correction of errors is desired in the application, then more testing needs to be done during production to characterize each accelerometer.

To measure and remove bias error and sensitivity error, do the following:

1. Place the sensor module on an approximately horizontal surface (table top, granite flat, etc.).
2. Rotate the sensor module 90 degrees to each of the six positions shown in Figure 2 below.
3. Read the outputs on each axis in each of the six positions.
4. Calculate the sensitivities (where the first subscript is the sensor, the second subscript is the position):

$$S_{xx} = \frac{(a_{x2} - a_{x4})}{2}; \qquad S_{yy} = \frac{(a_{y1} - a_{y3})}{2}; \qquad S_{zz} = \frac{(a_{z5} - a_{z6})}{2}$$

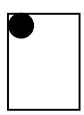
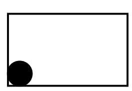
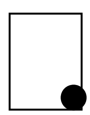
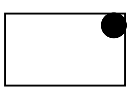
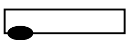
5. Calculate the sensor module biases:

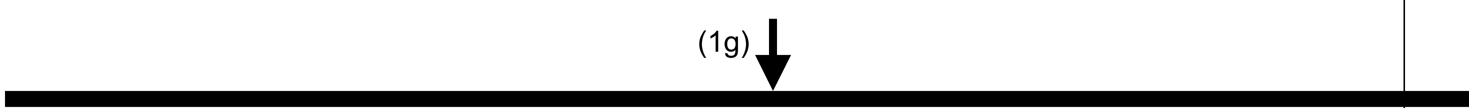
$$B_x^{0g} = \left(\frac{a_{x1} + a_{x3} + a_{x5} + a_{x6}}{4}\right); \quad B_y^{0g} = \left(\frac{a_{x2} + a_{x4} + a_{x5} + a_{x6}}{4}\right);$$

$$B_z^{0g} = \left(\frac{a_{x1} + a_{x2} + a_{x3} + a_{x4}}{4}\right)$$

6. Record B_x^{0g}, B_y^{0g}, B_z^{0g}, S_{xx}, S_{yy}, and S_{zz} in EEPROM or flash memory and use these values in all subsequent calculations of acceleration to get the corrected outputs.

Static X/Y/Z Output Response versus Orientation to Earth's surface (1g):

Position	1	2	3	4	5	6
Diagram					Top  Bottom	Bottom  Top
a_x	1.65 V	2.31 V	1.65 V	0.99 V	1.65 V	1.65 V
a_y	2.31 V	1.65 V	0.99 V	1.65 V	1.65 V	1.65 V
a_z	1.65 V	1.65 V	1.65 V	1.65 V	2.31 V	0.99 V
X-acceleration	**0**	**+1g**	**0**	**-1g**	**0**	**0**
Y-acceleration	**+1g**	**0**	**-1g**	**0**	**0**	**0**
Z-acceleration	**0**	**0**	**0**	**0**	**+1g**	**-1g**



Earth's Surface

Fig. 2) Rotating a sensor module to six different positions on a flat surface assuming a ±2g (0.66V/g) accelerometer operating at 3.3V.

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Temperature dependence

The output from the accelerometer varies with temperature. An accelerometer may have an offset that varies by 1 - 2.4mg/°C and a sensitivity that varies by 0.032%/°C. Choosing an accelerometer with small output variations over temperature is key to maintaining low errors, although this can incur extra cost for the component. Operating in an environment with small variations in temperature is another way to reduce errors. However, controlling the environment is difficult with portable handheld devices.

Temperature Compensation

Temperature compensation techniques are another way of reducing the errors even further. Because of slight variations in fabrication, every accelerometer is produced with a unique temperature characteristic. Since both the magnitude and the sign of the temperature coefficient are variable from unit to unit, the temperature compensation technique cannot simply consist of a temperature sensor in the feedback loop of an amplifier. Temperature mapping is needed. A temperature sensor is used to monitor temperature while the temperature is varied. The output of the accelerometer is measured and used to construct a look-up table or a formula that can be used to calculate a compensation factor. Usually the software used to construct the look-up table or formula is resident in the system microcontroller and the temperature sweeps are performed during system level testing or burn-in.

Ratiometric Error

Most MEMS accelerometers are essentially ratiometric. As the supply voltage varies (as batteries are discharging for example), the output voltages of the accelerometers scale proportionately. This phenomenon is known as ratiometricity. There can, however, be a slight error from ideal scaling with supply voltage. This error from ideal scaling is called ratiometric error. Formulas for calculating offset ratiometric error (ORE) and sensitivity ratiometric error (SRE) are shown below for the case of operation at a nominal supply voltage of 3.3V:

$$ORE(V_{dd}) = \left(\left(\frac{0gOffset(@V_{dd})}{0gOffset(@3.3V)}\right) - \frac{V_{dd}}{3.3V}\right) * 100$$

$$SRE(V_{dd}) = \left(\left(\frac{Sensitivity(@V_{dd})}{Sensitivity(@3.3V)}\right) - \frac{V_{dd}}{3.3V}\right) * 100$$

The larger the voltage swing, the larger the ratiometric error. For example, a voltage swing of 3.3V ± 10% (3.0V – 3.6V) could result in as much as 100 mg of error from the accelerometer. A voltage swing of 3.3V ± 5% (3.135V – 3.465V) for the same part would result in about 30 mg of error. Therefore, good voltage regulation is essential to reduce the source of this error.

Voltage Compensation

To further reduce ratiometric error, power supply voltage monitoring is needed. Similar to temperature compensation, the supply voltage can be monitored while the voltage is varied. The output of the accelerometer is measured and used to construct a look-up table or a formula that can be used to calculate a compensation factor. Once again, the software used to construct the look-up table or formula is resident in the system microcontroller and the voltage sweeps are performed during system level testing to construct a table similar to that shown in Figure 3. If system level testing is done with the sensor module in a static position, then only 0g offset corrections can be generated. If system level testing is done with the sensor module rotated to different positions (as shown in Figure 2), then 0g offset and sensitivity corrections can be generated.

Supply Voltage (V)	0g Offset (V)	Sensitivity (mV/g)
3.0	1.46	647
3.1	1.52	652
3.2	1.58	656
3.3	1.65	660
3.4	1.72	664
3.5	1.78	668
3.6	1.84	673

Fig. 3) Ratiometric voltage table assuming a ±2g (0.66V/g) accelerometer operating at 3.3V.

Subsequent calculations of accelerations are made based on what the supply voltage is currently measuring which determines the 0g offset and sensitivity values used in the calculations.

Non-linearity

Non-linearity is defined as the maximum deviation of the output response from a best fit line (-range to +range) expressed in a percentage of Full Scale Output (FSO).

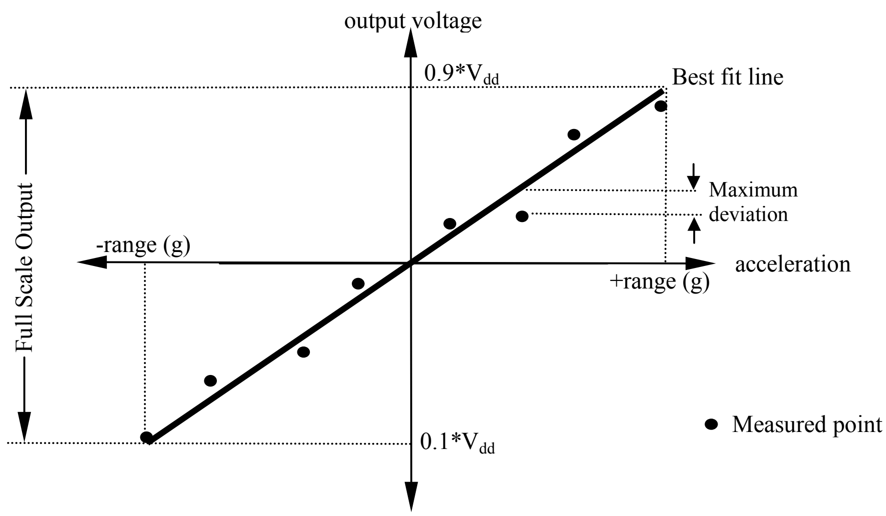

Fig. 4) Calculation of non-linearity

To measure non-linearity, the accelerometer is mounted on a linear shaker. The shaker vibrates with a controlled amplitude and frequency in a single direction. The accelerometer under test is mounted on the shaker with the axis to be measured parallel to the vibration direction. The output of the accelerometer under test and a reference accelerometer are recorded at 0.5g increments as the amplitude of the shaker increases from 0.5g to 5g. A least squares fit is performed to the data created by plotting the output voltage of the accelerometer versus the acceleration measured by the reference accelerometer. The deviation from ideal linearity is expressed in % of FSO.

$$Non-linearity = \frac{Maximum\,deviation\ (V)}{Full\ Scale\ Output\ (V)} \times 100\%$$

Procuring accelerometers of good linearity is very important. If tilt calibration is not provided with the accelerometer, a factory test must run through a robotic sequence of tilt angle tests, to correlate simulated versus measured pitch and roll angles to fill in the non-linear correction values. This becomes crucial when one degree accurate compassing demands better than 0.5 degree accurate pitch and roll angle measurement. Kionix accelerometers have extremely good linearity throughout the −1g (-90°) to +1g (+90°) range.

Cross Axis Sensitivity

Cross axis sensitivity is the variation in the accelerometer's output because of accelerations applied in axes perpendicular to the input axis of the accelerometer. For a tri-axis accelerometer, we can think of the accelerometers being characterized by a sensitivity matrix:

$$S = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix}$$

where the first subscript is the sensor measurement direction and the second subscript is the acceleration direction. For example, S_{xy} is the x-axis sensor's response to acceleration in the y-direction. The calculation of the cross axis sensitivity (expressed in percentage) for the x-axis sensor is shown below:

$$Sx_{cross} = \left(\frac{\sqrt{(S_{xy}^2 + S_{xz}^2)}}{S_{xx}} \right) * 100$$

The inherent cross axis sensitivity of the MEMS sensor is less than 0.15% based on its stiffness in each direction and its common mode rejection. The larger cross axis sensitivity that is actually observed is the by-product of positional inaccuracies at all stages of assembly. For Kionix's production testing, it's the sum of die placement error in the package, package placement in the test socket, socket placement on the test board, board placement inside test station, and test station alignment. For the customer, cross axis sensitivity will result from die placement error in the package, package placement on their board, board placement in their housing, and possibly, housing placement inside their application.

Adding the errors

What is the error in $Z = A + B$ where A and B are two measured quantities with errors ΔA and ΔB respectively?

At first one might think that the error in Z would be just the sum of the errors in A and B.

$$(A+\Delta A)+(B+\Delta B)=(A+B)+(\Delta A+\Delta B)$$

and

$$(A-\Delta A)+(B-\Delta B)=(A+B)-(\Delta A+\Delta B).$$

But this assumes that, when combined, the errors in A and B have the same sign and maximum magnitude; that is that they always combine in the worst possible way. This could only happen if the errors in the two variables were perfectly correlated, i.e. if the two variables were not really independent.

If the variables are independent then sometimes the error in one variable will happen to cancel out some of the error in the other and so, on the average, the error in Z will be less than the sum of the errors in its parts. A reasonable way to try to take this into account is to treat the perturbations in Z produced by perturbations in its parts as if they were "perpendicular" and added according to the Pythagorean theorem (geometric sum),

$$(A\pm\Delta A)+(B\pm\Delta B)=(A+B)\pm\sqrt{(\Delta A)^2+(\Delta B)^2}$$

When calculating the error in the acceleration measured by the accelerometer, remember to use the operating temperature and voltage to calculate the temperature and ratiometric effects.

Noise

Accuracy refers to how closely a measured value agrees with the correct value, in this case, the correct acceleration. **Precision** refers to how closely individual measurements agree with each other. So far, this application note has talked only about errors which affect the accuracy of the accelerometer output. Now we'll discuss errors that affect the precision of the output.

Basically, these errors fall into the category labeled as noise. There are two types of noise in the accelerometer: the electronic noise from the circuitry which is converting the motion into a voltage signal and the mechanical noise from the sensor itself. There are several sources of electronic noise – Johnson noise, shot noise, flicker noise, etc. – which are discussed in detail in many electronic and electrical engineering textbooks. The ASIC inside Kionix's accelerometer products has been designed to reduce these sources of noise as much as possible. The mechanical noise of the sensor comes from thermo-mechanical noise and environmental vibrational noise.

Thermo-mechanical noise derives from the fact that MEMS accelerometers are comprised of small moving parts. These small parts are susceptible to mechanical noise resulting from molecular agitation. Fundamentally, the magnitude of the thermo-mechanical noise density ($ND_{thermo\text{-}mech}$) depends on the the resonant frequency ω, mass m, damping Q, and temperature T of the sensor.

$$ND_{thermo-mech} = \frac{\sqrt{\frac{4k_B T\omega}{mQ}}}{g} \quad in\ units\ of\ {}^{g}\!/\!_{\sqrt{Hz}}$$

Thermo-mechanical noise is often one of the limiting noise components of MEMS accelerometers. As with the electronic noise of the ASIC, good design practices of the sensor reduce the thermo-mechanical noise as low as possible.

Environmental noise is one of the largest noise sources in accelerometer measurements, especially sensitive low-g measurements. Accelerometers are good instruments for measuring vibrations. It's one of the many applications that accelerometers are used for. However, in most handheld applications, tilt or simple linear accelerations are the quantities to be measured, not the vibrations or tremors of your hand. As an example, hold a laser pointer in one hand and point it at a wall several meters away to see how steady your hand is. The vibrations of your hand make the accelerometer outputs appear "jittery" or the application "jumpy." Other sources of vibrational noise can be fan motors, speakers, or disk drives. Care must sometimes be taken when placing a sensitive accelerometer in close proximity to these items.

Reducing the noise on the accelerometer outputs can be done through low pass filtering. There are a great many different types of filter circuits (Butterworth, Chebyshev, and Bessel, to name a few), with different responses to changing frequency. The easiest filter to create is a simple RC low pass filter with the –3dB point set at the bandwidth you want. As an example, the application circuit of the KXPA4 shows how to connect capacitors on the outputs of a Kionix accelerometer to form a first order low pass filter.

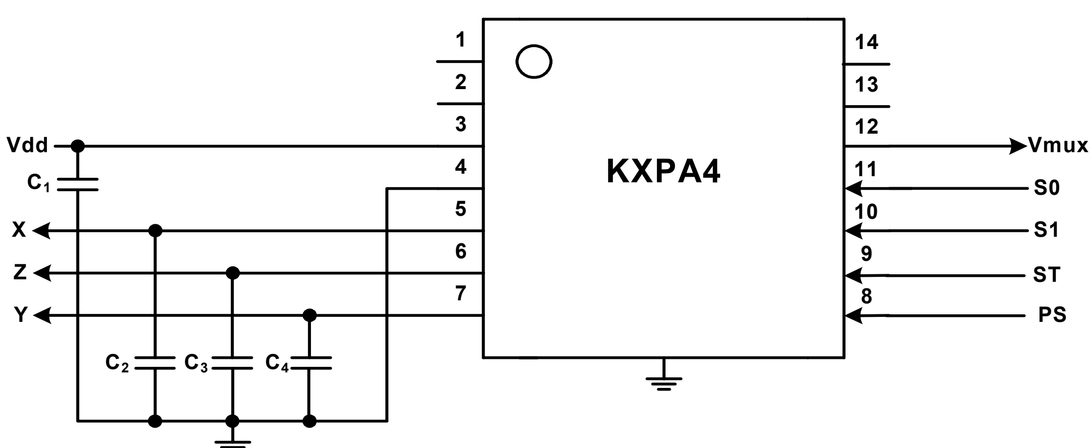

Fig. 6) KXPA4 application circuit with low pass filters

The bandwidth is determined by the filter capacitors connected from pins 5, 6 and 7 to ground. Given a desired bandwidth, f_{BW}, the filter capacitors are determined by:

$$C_2 = C_3 = C_4 = \frac{4.97 * 10^{-6}}{f_{BW}} \quad F$$

A first-order filter, a simple RC filter like the one shown above, will reduce the signal amplitude by half (about –6 dB) every time the frequency doubles (goes up one octave). A second-order filter does a better job of attenuating higher frequencies. For example, a second-order Butterworth filter will reduce the signal amplitude to one fourth its original level every time the frequency doubles (–12 dB per octave). Third- and higher-order filters are defined similarly. In general, the final rate of rolloff for an n^{th}-order filter is -6n dB per octave.

For accurate noise measurements, you need to know the equivalent noise bandwidth. This noise bandwidth, B, is what is used in the calculation of the noise amplitude from the noise density. For the first order simple RC low pass filter:

$$B = 1.57 f_{-3dB} \quad Hz$$

For other filters, the noise bandwidth will have other factors. For example, a Butterworth filter will give the following noise bandwidths:

$$B = 1.57 f_{-3dB} \quad Hz \qquad (1^{st}\ order)$$

$$B = 1.11 f_{-3dB} \quad Hz \qquad (2^{nd}\ order)$$

$$B = 1.05 f_{-3dB} \quad Hz \qquad (3^{rd}\ order)$$

$$B = 1.025 f_{-3dB} \quad Hz \qquad (4^{th}\ order)$$

To calculate the noise (or resolution) of an accelerometer in the absence of environmental vibration, one uses the noise density (ND) parameter. This parameter can be found in Figure 5 above for a variety of Kionix accelerometers. Technically, this is an RMS noise-acceleration density – similar to an RMS noise-voltage density. The following equation is used to calculate the RMS acceleration noise (a_n) you would measure in a noise bandwidth B:

$$a_n = ND * \sqrt{B}$$

As an example, the RMS acceleration noise you would measure with a KXPS5 with a 50Hz first order low pass filter would be:

$$a_n = 250 \, {}^{\mu g}\!/\!_{\sqrt{Hz}} * \sqrt{1.57(50 Hz)} = 2215 \, \mu g_{rms} = 2.215 \, mg_{rms}$$

This noise could be reduced by placing a higher order filter on the outputs or by reducing the bandwidth of the filter. Of course, one of the penalties for placing a higher order filter on the outputs is the number of external components required on the printed circuit board. One of the penalties for reducing the bandwidth is the increase in startup/response time of the outputs. Depending on the application, this may result in a sluggish response of the application to motion, resulting in a poor experience for the user.

Another technique that can be used to reduce noise is over-sampling and averaging. In most applications, digital data from the accelerometer is used in computations or in control functions. This digital data is either obtained directly from the accelerometer or after the analog data from the accelerometer is passed through an analog to digital converter (ADC). In either case, the system is acquiring accelerometer information at a particular sampling frequency. For a cell phone screen rotation application, the sampling rate may be 10 Hz while a hard drive protection application may require 1000 Hz.

The required sampling frequency in accordance with the Nyquist Theorem is the Nyquist frequency (f_N):

$$f_N = 2 * f_{-3dB}$$

where f_{-3dB} is the low pass filter cutoff frequency. Oversampling is the process of sampling a signal with a sampling frequency significantly higher than the Nyquist frequency. For example, the cell phone screen rotation may be acquiring data at a 100 Hz sampling frequency. Every 10 samples are averaged together, and that average value is reported at a 10 Hz frequency and used in the application to determine if the screen has rotated. In this way, the noise in the acceleration signal is reduced. If multiple samples (N) are taken of the same quantity with a random noise signal, then averaging those samples reduces the noise by a factor of $1/\sqrt{N}$.

Conclusion

There are many ways that errors can enter into an accelerometer's measurement which will affect its accuracy and precision. Depending on the requirements of the application for which the measurements will be used, techniques can be utilized to reduce or eliminate these errors. A few of those techniques have been discussed, and many more techniques are discussed in literature. Obviously test costs and time must be weighed against the desire or need for accuracy and precision in the application.

Theory of Operation

Kionix MEMS linear tri-axis accelerometers function on the principle of differential capacitance. Acceleration causes displacement of a silicon structure resulting in a change in capacitance. A signal-conditioning CMOS technology ASIC detects and transforms changes in capacitance into an analog output voltage which is proportional to acceleration. These outputs can then be sent to a micro-controller for integration into various applications. Kionix technology provides for X, Y and Z-axis sensing on a single, silicon chip. One accelerometer can be used to enable a variety of simultaneous features including, but not limited to:

- Drop force modeling for warranty management
- Hard disk drive shock protection
- Tilt screen navigation
- Theft, man-down, accident alarm

- Image stability, screen orientation
- Computer pointer
- Navigation, mapping
- Game playing

For product summaries, specifications, and schematics, please refer to the Kionix accelerometer product sheets at http://www.kionix.com/sensors/accelerometer-products.html.