



## Using Two Tri-Axis Accelerometers for Rotational Measurements

### Introduction

In many applications, customers would like to measure rotational motions (angular velocity, angular acceleration) in addition to linear motions. Most often, gyroscopes are added to their end product to obtain the rotational information. In some instances, customers already have a system that contains two or more accelerometers. With some understanding of fundamental physics, they can extract more than just linear acceleration data from their system.

This application note describes how to use two Kionix MEMS low-g accelerometers to enable rotational measurements. Applicable theory, plots and equations are provided with this note as guidelines.

### The Theory

Two accelerometers are rigidly mounted on a spinning object at distances  $r_1$  and  $r_2$  from the center of rotation as shown in Figure 1.

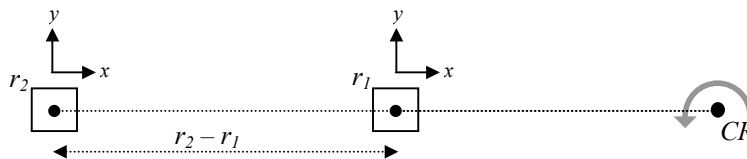


Fig. 1) Locations of two accelerometers on a rotating object

The radial acceleration measured by accelerometer 1 is:

$$a_{x1} = \omega^2 r_1 \quad (1)$$

where  $\omega$  is the angular velocity. The radial acceleration measured by accelerometer 2 is:

$$a_{x2} = \omega^2 r_2 \quad (2)$$

Taking the difference between the two measurements:

$$a_{x2} - a_{x1} = \omega^2 (r_2 - r_1) = \omega^2 D \quad (3)$$

where  $D = r_2 - r_1$  is the fixed separation between the two accelerometers. From this equation the magnitude of the angular velocity can be determined:

$$\omega = \sqrt{\frac{|a_{x2} - a_{x1}|}{D}} \quad (4)$$

The tangential acceleration measured at accelerometer 1 is:

$$a_{y1} = \alpha r_1 \quad (5)$$

where  $\alpha$  is the angular acceleration. The tangential acceleration measured at accelerometer 2 is:

$$a_{y2} = \alpha r_2 \quad (6)$$

Taking the difference between the two measurements:

$$a_{y2} - a_{y1} = \alpha(r_2 - r_1) = \alpha D \quad (7)$$

Re-arranging the terms to determine the angular acceleration:

$$\alpha = \frac{(a_{y2} - a_{y1})}{D} \quad (8)$$

Looking at Eqns. (4) and (8), a couple of observations can be made. First, it is not necessary to know where the center of rotation is to determine the angular velocity or angular acceleration. The separation between the accelerometers, established at the design of the system, is the only parameter needed. Second, this separation and the resolution of the accelerometers determines the resolution that can be obtained in the angular measurements. For accelerometers with a given resolution, better resolution of the angular measurements can be obtained by separating the accelerometers by a larger distance.

Now consider two tri-axis accelerometers separated by distance  $D$  with X-axes aligned as shown in Figure 2. The radial and tangential accelerations should be able to determine angular velocity around the Y-axis ( $\omega_y$ , roll rate) and the Z-axis ( $\omega_z$ , yaw rate).

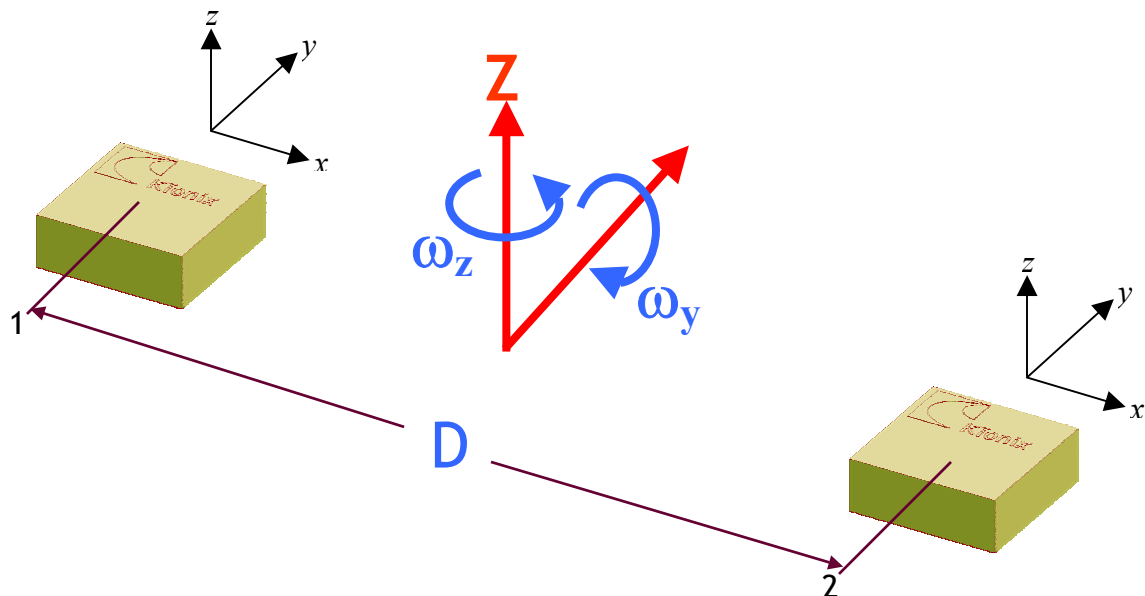


Fig. 2) Two tri-axis accelerometers on a rotating rigid body

## The Algorithm

To calculate the rotational rates, we start by sampling the X-, Y-, and Z-axis accelerations of both accelerometer #1 and accelerometer #2. Keeping a running average (around 5 samples at 60 Hz) of the accelerations will reduce noise. The total rotation rate magnitude ( $\omega$ ) is calculated directly from radial acceleration as shown in Eqn. 4. Any  $\omega$  that is below a threshold ( $\omega_t$ ) is set to 0.

$$\omega = \sqrt{\frac{|a_{x2} - a_{x1}|}{D}} \text{ when } \omega \geq \omega_t$$

$$\omega = 0 \text{ when } \omega < \omega_t$$
(9)

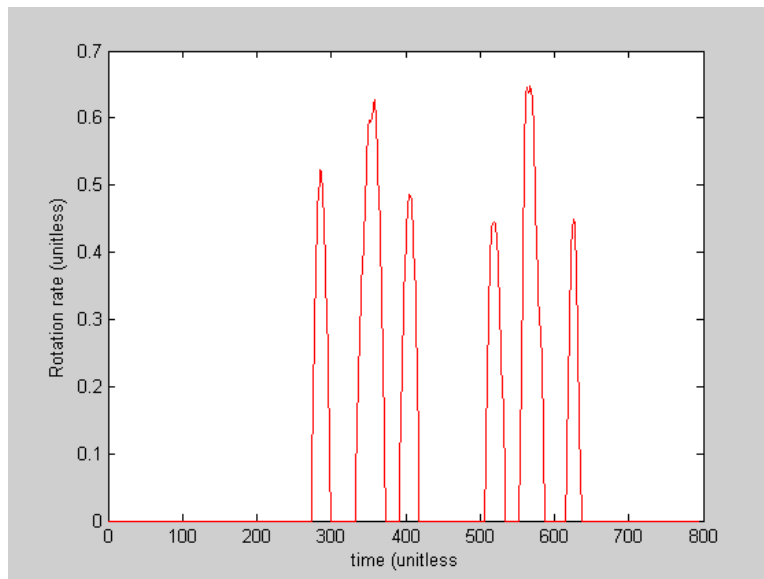


Fig. 3) An example of the total rotation rate magnitude calculated using Eqn. 9.

The total rotation rate is now known, but the direction (sign) and the rotation rate about the Y-axis (roll) and the Z-axis (yaw) are still unknown. Using Eqn. 8, the angular accelerations about the z-axis and y-axis are calculated from the Y-axis and Z-axis accelerations:

$$\alpha_z = \frac{(a_{y2} - a_{y1})}{D} \quad (10)$$

$$\alpha_y = \frac{(a_{z2} - a_{z1})}{D} \quad (11)$$

Integrating  $\alpha_y$  and  $\alpha_z$  when  $\omega$  is nonzero allows us to determine  $\omega_y$  and  $\omega_z$ . Thus, the relative magnitude and direction of rotation are calculated from the integrals of the tangential accelerations.

$$\omega_z = \int_0^t \alpha_z dt \quad (12)$$

$$\omega_y = \int_0^t \alpha_y dt \quad (13)$$

Since the angular accelerations,  $\alpha_y$  and  $\alpha_z$ , rise before  $\omega$ , it is necessary to add the integral of  $\alpha_y$  and  $\alpha_z$  from several time steps before  $\omega$  crosses the  $\omega_t$  threshold.

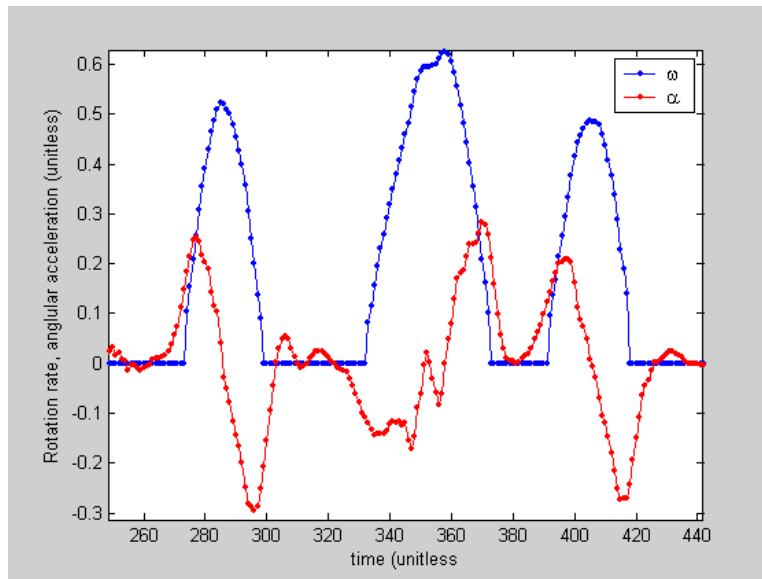


Fig. 4) Timing of  $\alpha$  and  $\omega$  showing  $\alpha$  preceding  $\omega$ .

The vector sum of  $\omega_y$  and  $\omega_z$  from Eqns. 12 and 13 is equal to the magnitude of the total angular rotation rate,  $\omega_{total}$ .

$$\omega_{total} = \sqrt{\omega_y^2 + \omega_z^2} \quad (14)$$

Using the total rotation rate magnitude calculated in Eqn. 9 and the relative magnitude and direction of rotation obtained from Eqn. 12, we can determine the yaw:

$$yaw = \omega * \frac{\omega_z}{\omega_{total}} \quad (15)$$

and the roll:

$$roll = \omega * \frac{\omega_y}{\omega_{total}} \quad (16)$$

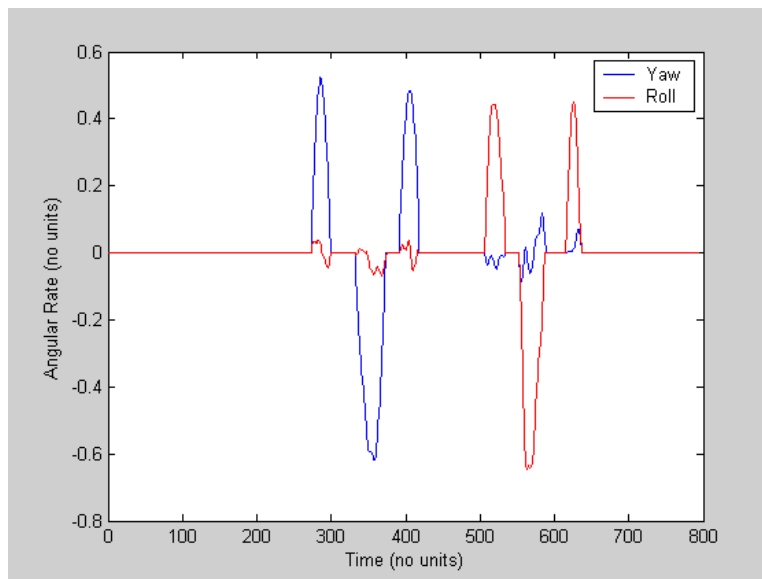


Fig. 5) Examples of yaw and roll rates calculated by Eqns. 15 and 16.

## Test Cases

To test the algorithm, Kionix mounted two KXP74-2050 accelerometers separated by a distance of 9.2cm on a rigid body. A gyro was also mounted on the rigid body so that the angular rate determined by the two accelerometers could be directly compared to the angular rate output of the gyro. The following plots show the results of the testing.

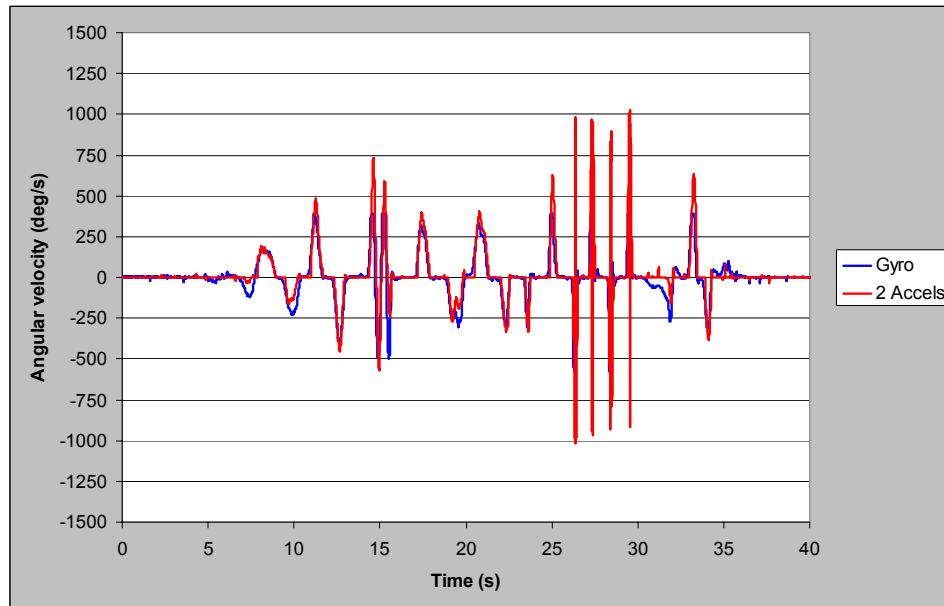


Fig. 6a) Measurement of angular velocity using a gyro and 2 accelerometers.

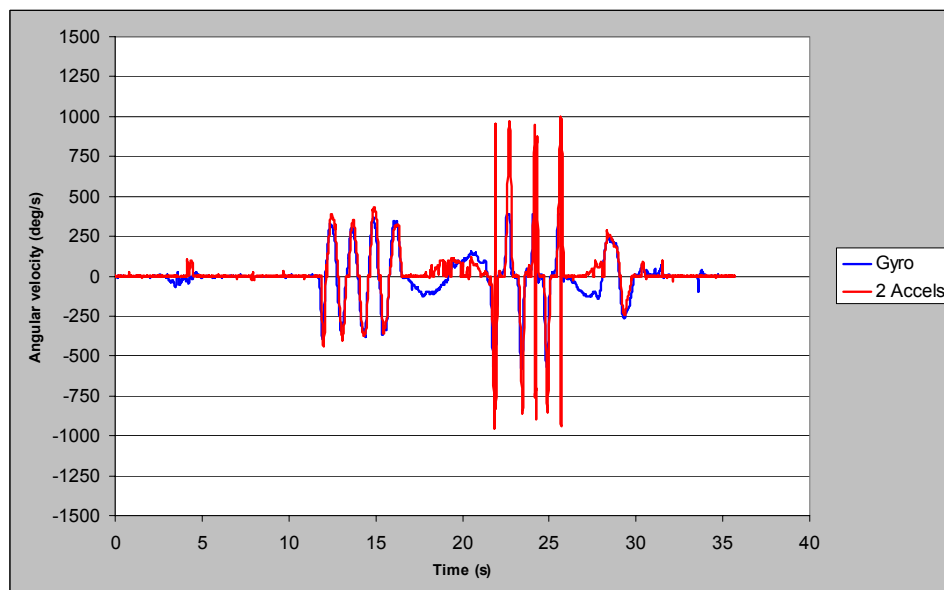


Fig. 6b) Another measurement of angular velocity using a gyro and 2 accelerometers.

As seen in the measurements, this method works very well for certain motions – short (<2 seconds), medium magnitude (140 °/sec to 1200 °/sec) rotations. Low magnitude (<140 °/sec) motions are not sensed because they are below the system

resolution. During long motions (>2 seconds), the integration error becomes large. Very large magnitude (>1200 °/sec) motions will cause error because the accelerometer outputs saturate.

We also found that the accelerometer sensitivities had to be well matched and any offset errors had to be corrected through a start-up calibration.

Source code is available for running this algorithm using a Silabs 8051F312 and two Kionix KXP74-2050 accelerometers. Send an e-mail to [rotation@kionix.com](mailto:rotation@kionix.com) for more information.

## Conclusions

The measurements obtained in this study verified two linear accelerometers can be used to determine angular rotation rates. This method works best when the rotational motions are quick with large angular accelerations. In this case, there is no chance of dividing by zero. Integration only takes place for short time, so drift is not a problem. Angular accelerations need to be lower than the accelerometer sensing range.

This method does not work as well when angular accelerations ( $\alpha_y$  and  $\alpha_z$ ) are very small, since the algorithm relies on knowing the sign confidently. There is some sensitivity to offset and sensitivity differences between the two sensors. Therefore, some method is needed to compensate for sensor mismatch, such as a calibration on start-up. Selection of the  $\omega$  threshold ( $\omega_t$ ) is dependent on sensor performance and the expected motions in the application.

Using two accelerometers in this manner can work well in applications where the expected motions align with the capabilities. Calculation of angular rate with 2 accelerometers can be performed in existing microprocessors and may provide significant cost and power consumption advantages over gyro solutions.

## The Kionix Advantage

Kionix technology provides for X, Y, and Z-axis sensing on a single, silicon chip. One accelerometer can be used to enable a variety of simultaneous features including, but not limited to:

- Hard Disk Drive protection
- Vibration analysis
- Tilt screen navigation
- Sports modeling
- Theft, man-down, accident alarm
- Image stability, screen orientation & scrolling
- Computer pointer
- Navigation, mapping
- Game playing
- Automatic sleep mode

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## Theory of Operation

Kionix MEMS linear tri-axis accelerometers function on the principle of differential capacitance. Acceleration causes displacement of a silicon structure resulting in a change in capacitance. A signal-conditioning CMOS technology ASIC detects and transforms changes in capacitance into an analog output voltage, which is proportional to acceleration. These outputs can then be sent to a micro-controller for integration into various applications. For product summaries, specifications, and schematics, please refer to the Kionix MEMS accelerometer product sheets at <http://www.kionix.com/sensors/accelerometer-products.html>.